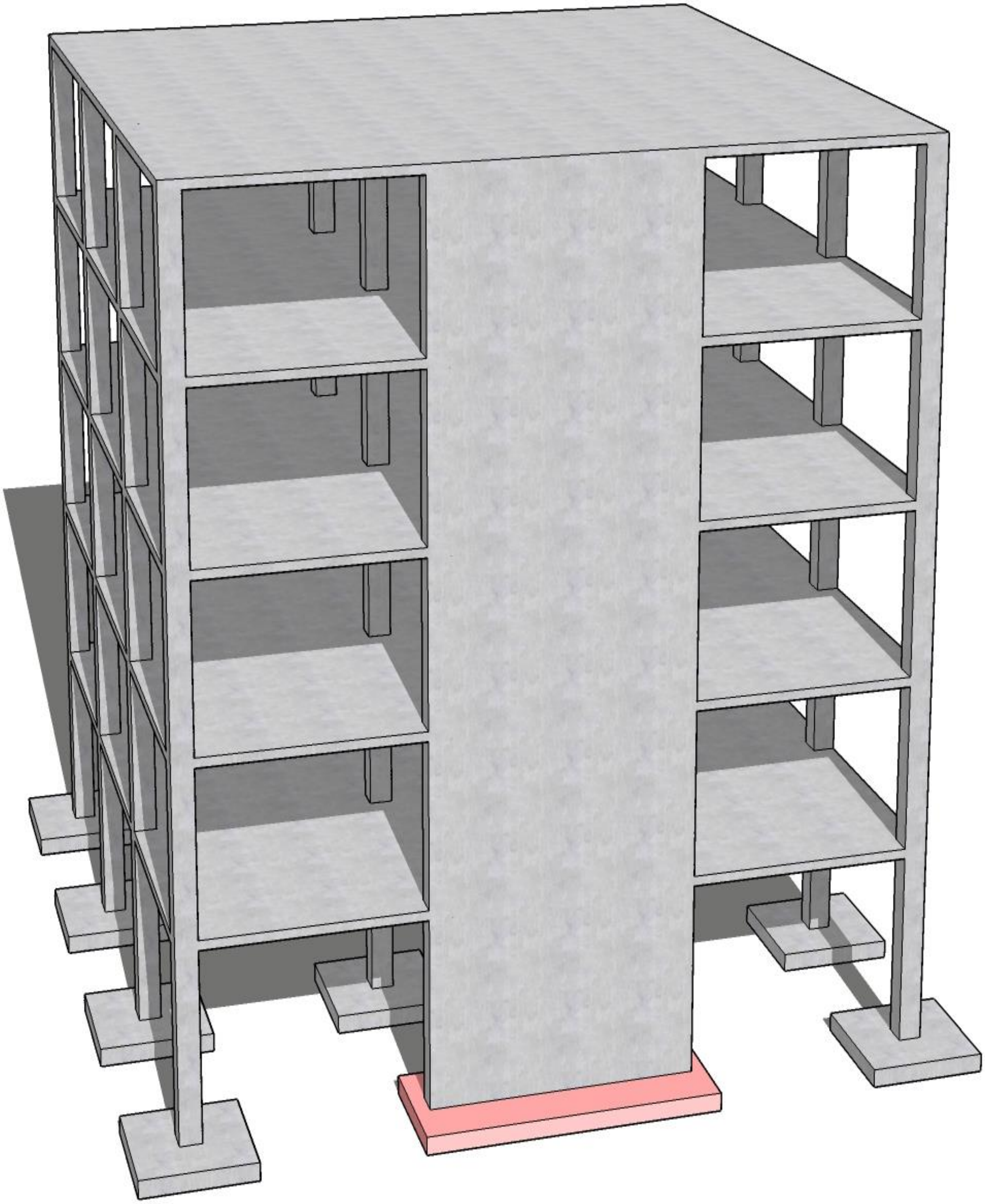
Reinforced Concrete Shear Wall Foundation (Strip Footing) Analysis and Design



Reinforced Concrete Shear Wall Foundation (Strip Footing) Analysis and Design

A 12 in. thick structural reinforced concrete shear wall is to be supported by a strip footing. The shear wall carries service dead and live loads of 10 kips/ft and 12.5 kips/ft respectively. The allowable soil pressure is 5000 psf. The wall footing is to be based 5 ft below the final ground surface. Design the footing for flexure, shear and allowable soil pressure.

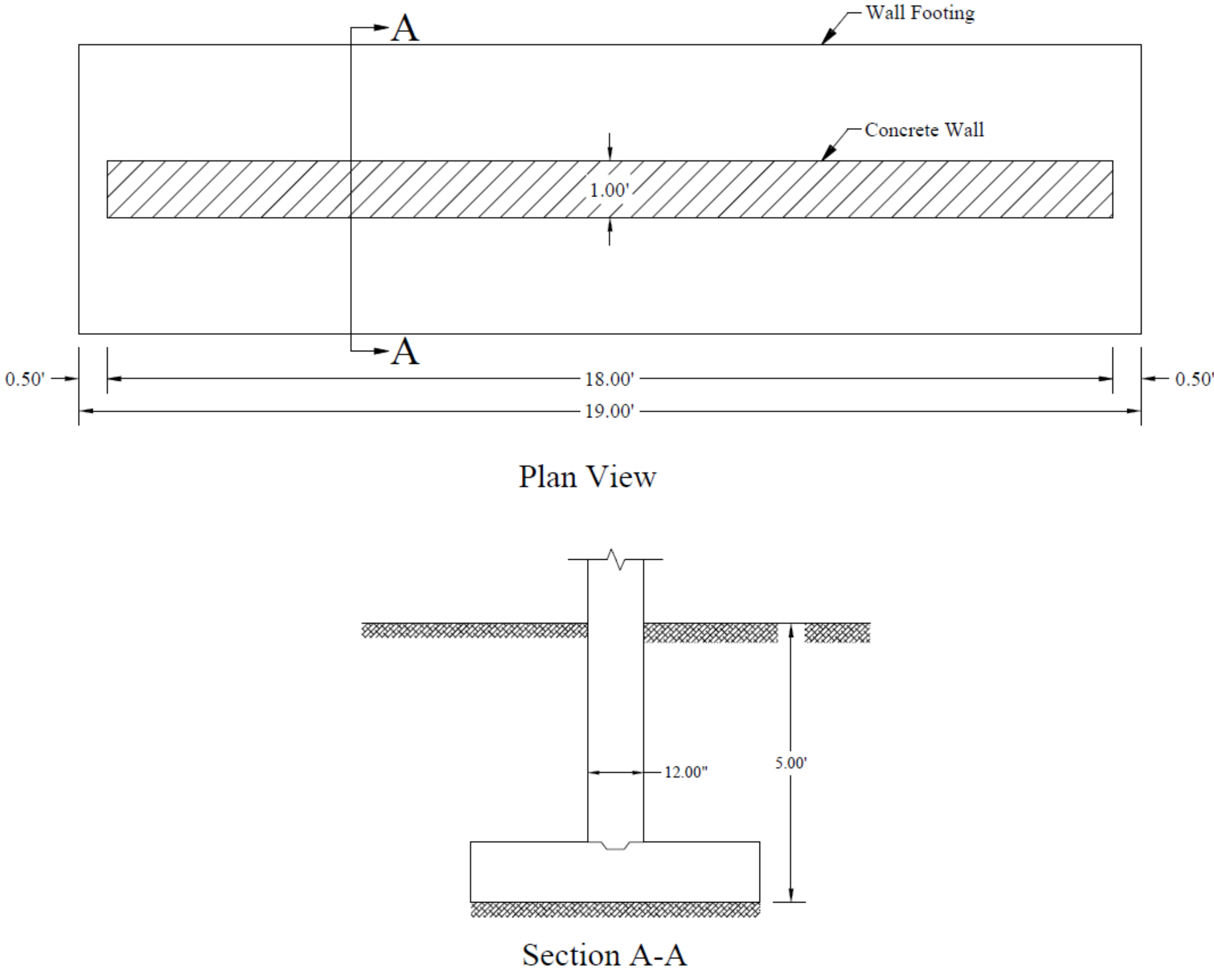


Figure 1 – Reinforced Concrete Wall Footing Geometry

Contents

Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

Reference

Reinforced Concrete Mechanics and Design, 7th Edition, 2016, James Wight, Pearson, Example 15-1

spMats Engineering Software Program Manual v8.12, StucturePoint LLC., 2016

Design Data

f_c' = 3,000 psi normal weight concrete

f_y = 60,000 psi

Wall thickness = 12 in.

Distance from the ground level to the footing base = 5 ft

Dead load, D = 10 kips/ft

Live load, L = 12.5 kips/ft

Soil density, γ_s = 120 pcf

Concrete density, γ_c = 150 pcf for normal weight concrete

Allowable soil pressure, q_a = 5000 psf

1. Preliminary Member Sizing

1.1. Footing Cross Sectional Dimensions

In order to calculate the allowable net soil pressure, it is necessary to guess the footing thickness for a first trial in order to estimate the footing self-weight. Generally footing thickness of 1 to 1.5 times the wall thickness will be adequate. Assuming the footing thickness is equal to the thickness of the wall (t_f = 12 in.).

The allowable net soil pressure is equal to the allowable soil pressure minus the self-weight of the footing and soil weight over the footing:

$$q_n = q_a - weight_{footing} - weight_{soil} = 5\text{ ksf} - \left(1\text{ ft}\times 0.15\text{ kcf}\right) - \left(\left(5\text{ ft} - 1\text{ ft}\right)\times 0.12\text{ kcf}\right) = 4.37\text{ ksf}$$

This value is the balance of allowable soil pressure available to resist applied loads (dead, live, etc.) from the wall. Estimate the minimum base area of foundation based on unfactored forces and moments transmitted by wall foundation to soil. ***ACI 318-14 (13.3.1.1)***

$$A_{required} = \frac{P_{service}}{q_n} = \frac{10+12.5}{4.37} = 5.15\text{ ft}^2$$

Considering a 1 ft strip of wall and footing, the minimum footing width is 5.15 *ft*. Try 5.17 *ft* (5 *ft* 2 *in*.).

1.2. Factored Net Pressure

The factored net pressure that will be used in the design of the concrete and reinforcement is equal to:

$$q_{nu} = \frac{P_u}{A_{footing}} = \frac{1.2\times 10 + 1.6\times 12.5}{1\times 5.17} = 6.19\text{ ksf}$$

2. Shear Capacity Check

For this type of foundation, one-way shear is dominant in comparison with two way shear and is therefore a significant design parameter. The critical section for one-way shear is located at distance *d* from the face of the wall.

$$d = t_f - \text{cover} - d_b / 2 = 12 - 3 - 0.5/2 = 8.75 \text{ in.}$$

$$V_u = q_{nu} \times A_{tributary} = 6.19 \times (16.25/12 \times 1) = 8.38 \text{ kips/ft}$$

$$\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f_c'} \times b_w \times d$$ ***ACI 318-14 (22.5.5.1)***

$$\phi V_c = 0.75 \times 2 \times 1.0 \times \sqrt{3000} \times 12 \times 8.38 / 1000 = 8.26 \text{ kips/ft}$$

Where ϕ = 0.75 ***ACI 318-14 (Table 21.2.1)***

$V_u > \phi V_c$ → Thicker footing is required, try 13 in.

$$d = t_f - \text{cover} - d_b / 2 = 13 - 3 - 0.5/2 = 9.75 \text{ in.}$$

$$V_u = q_{nu} \times A_{tributary} = 6.19 \times (15.25/12 \times 1) = 7.87 \text{ kips/ft}$$

$$\phi V_c = \phi \times 2 \times \lambda \times \sqrt{f_c'} \times b_w \times d$$ ***ACI 318-14 (22.5.5.1)***

$$\phi V_c = 0.75 \times 2 \times 1.0 \times \sqrt{3000} \times 12 \times 9.75 / 1000 = 9.61 \text{ kips/ft}$$

$V_u < \phi V_c$ → o.k.

∴ use footing with 13 in. thick and 5 ft 2 in. wide.

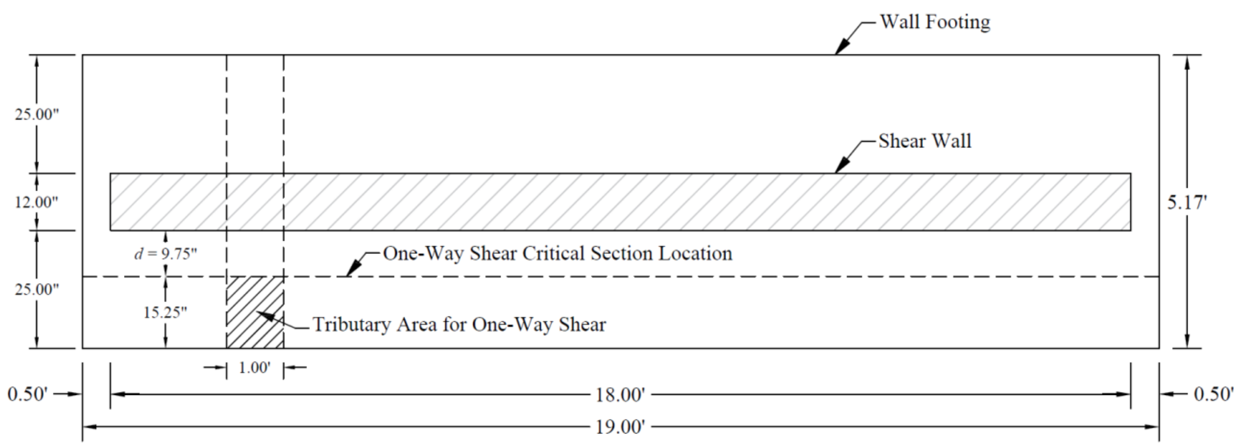


Figure 2 – Strip Footing Plan Showing Tributary Area for One-Way Shear

3. Flexural Reinforcement Design

The critical section for moment is at the face of the wall. The design moment is:

$$M_u = q_{nu} \times A_{tributary} \times \frac{l_{tributary}}{2} = 6.19 \times (25/12 \times 1) \times \frac{25/12}{2} = 13.4 \text{ kip-ft/ft}$$

Use d = 9.75 in.

To determine the area of steel, assumptions have to be made whether the section is tension or compression controlled, and regarding the distance between the resultant compression and tension forces along the footing section (jd). In this example, tension-controlled section will be assumed so the reduction factor ϕ is equal to 0.9, and jd will be taken equal to $0.95d$. The assumptions will be verified once the area of steel in finalized.

Assume $jd = 0.95 \times d = 9.26$ in.

$$A_s = \frac{M_u}{\varphi f_y jd} = \frac{13.4 \times 12000}{0.9 \times 60000 \times 9.26} = 0.321 \text{ in.}^2/\text{ft}$$

Recalculate $'a'$ for the actual $A_s = 0.321 \text{ in.}^2/\text{ft} \rightarrow a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.321 \times 60000}{0.85 \times 3000 \times 12} = 0.629$ in.

$$c = \frac{a}{\beta_1} = \frac{0.629}{0.85} = 0.74 \text{ in.}$$

$$\varepsilon_t = \left(\frac{0.003}{c}\right) d_t - 0.003 = \left(\frac{0.003}{0.74}\right) \times 9.75 - 0.003 = 0.037 > 0.005$$

Therefore, the assumption that section is tension-controlled is valid.

$$A_s = \frac{M_u}{\phi f_y (d - a/2)} = \frac{13.4 \times 12000}{0.9 \times 60000 \times (9.75 - 0.629/2)} = 0.316 \text{ in.}^2/\text{ft}$$

$$A_{s,\min} = \text{Greater of} \begin{Bmatrix} \dfrac{0.0018 \times 60,000}{f} \\ 0.0014 \end{Bmatrix} \times b \times h$$ **ACI 318-14 (7.6.1.1)**

$$A_{s,\min} = 0.0018 \times 12 \times 13 = 0.281 \text{ in.}^2/\text{ft} < 0.316 \text{ in.}^2/\text{ft}$$

$$s_{\max} = \text{ lesser of } \begin{Bmatrix} 3h \\ 18 \text{ in.} \end{Bmatrix} = \text{lesser of } \begin{Bmatrix} 3 \times 13 = 39 \text{ in.} \\ 18 \text{ in.} \end{Bmatrix} = 18 \text{ in.}$$ **ACI 318-14 (7.7.2.3)**

Provide #4 bars at 7 in. on centers ($\leq s_{max}$) with A_s = 0.34 in.^{2}/ft. Note that #5 bars at 11 in. on centers with A_s = 0.34 in.^{2}/ft can be also used. Reinforcement along the wall length is governed by shrinkage and temperature requirements and is detailed below.

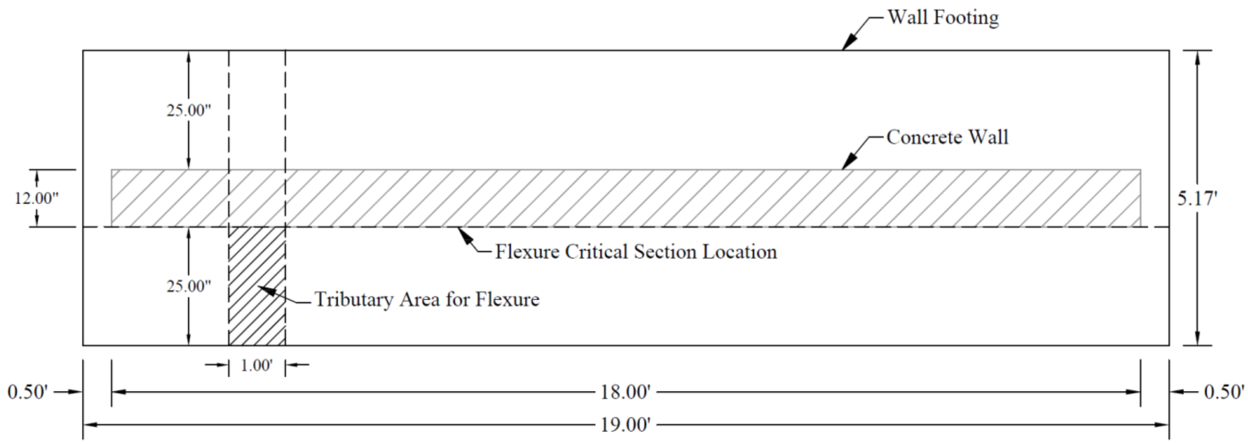


Figure 3 – Strip Footing Plan Showing Tributary Area for Flexure

4. Reinforcement Detailing

4.1. Development Length

Check if the simplified development length equation can be used:

Bars used are #4 (< #6)

Clear spacing of the bars being developed exceeds $2d_b$ (7 – 0.5 = 6.5 in. > 2 x 0.5 = 1.0 in.)

Clear cover exceeds d_b (3 – 0.5 = 2.5 in. > 0.5 in.)

Use the simplified equation:

$$l_d = \left(\frac{f_y \Psi_t \Psi_e}{25\lambda\sqrt{f_c^{'}}} \right) d_b = \left(\frac{60,000 \times 1.0 \times 1.0}{25 \times 1.0 \times \sqrt{3000}} \right) \times 0.5 = 21.9 \text{ in.}$$

ACI 318-14 (Table 25.4.2.2)

Where:

$\lambda = 1.0$ (Light weight modification factor: normal weight concrete) ***ACI 318-14 (Table 25.4.2.4)***

$\Psi_t = 1.0$ (Casting position modification factor: less than 12 in. of fresh concrete placed below horizontal reinforcement) ***ACI 318-14 (Table 25.4.2.4)***

$\Psi_e = 1.0$ (Epoxy modification factor: uncoated or zinc-coated reinforcement) ***ACI 318-14 (Table 25.4.2.4)***

The provided bar length is equal to:

$l_{d,provided} = l_{tributary} - \text{cover} = 25 - 3 = 22 \text{ in.} \geq l_d = 21.9 \text{ in.}$ → o.k.

4.2. Shrinkage and Temperature Reinforcement

Shrinkage and temperature reinforcement is checked along the length of the footing and is calculated as follows:

$$A_{s,shrinkage} = \text{Greater of} \begin{Bmatrix} \dfrac{0.0018 \times 60,000}{f} \\ 0.0014 \end{Bmatrix} \times b \times h$$ **ACI 318-14 (24.4.3.2)**

$$A_{s,shrinkage} = 0.0018 \times 62 \times 13 = 1.45 \text{ in.}^2$$

$$s_{max} = \text{ lesser of } \begin{Bmatrix} 5h \\ 18 \text{ in.} \end{Bmatrix} = \text{lesser of} \begin{Bmatrix} 5 \times 13 = 65 \text{ in.} \\ 18 \text{ in.} \end{Bmatrix} = 18 \text{ in.}$$ **ACI 318-14 (24.4.3.3)**

Provide 5-#5 bars at 13.84 in. on centers ($\leq s_{max}$) with A_s = 1.55 in.^{2} (Note that 3-#7 bars at 18 in. on centers with A_s = 1.80 in.^{2} can be also used).

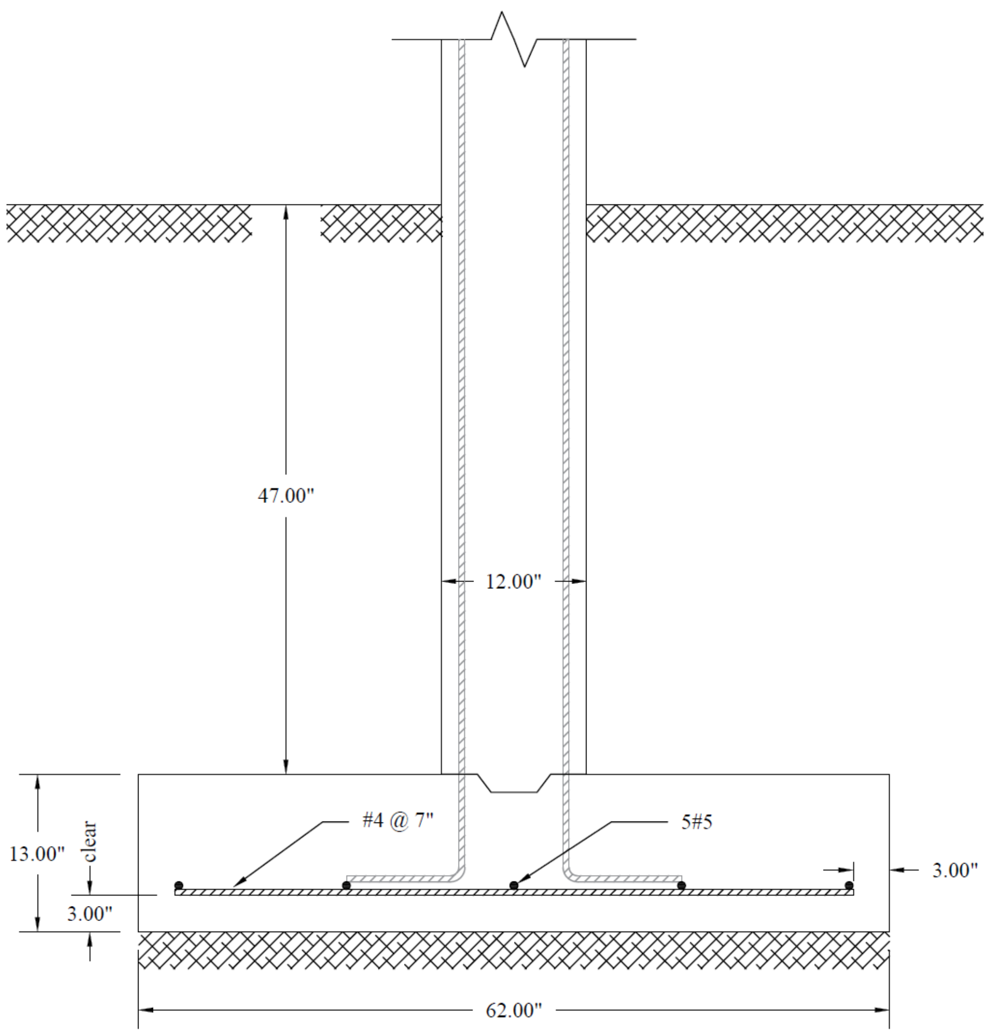


Figure 4 – Wall Footing Reinforcement Details

5. Strip Footing Analysis and Design – spMats Software

spMats uses the Finite Element Method for the structural modeling and analysis of reinforced concrete slab systems or mat foundations subject to static loading conditions.

The slab, mat, or footing is idealized as a mesh of rectangular elements interconnected at the corner nodes. The same mesh applies to the underlying soil with the soil stiffness concentrated at the nodes. Slabs of irregular geometry can be idealized to conform to geometry with rectangular boundaries. Even though slab and soil properties can vary between elements, they are assumed uniform within each element.

For illustration and comparison purposes, the following figures provide a sample of the input modules and results obtained from an spMats model created for the reinforced concrete strip footing (shear wall foundation) in this example.

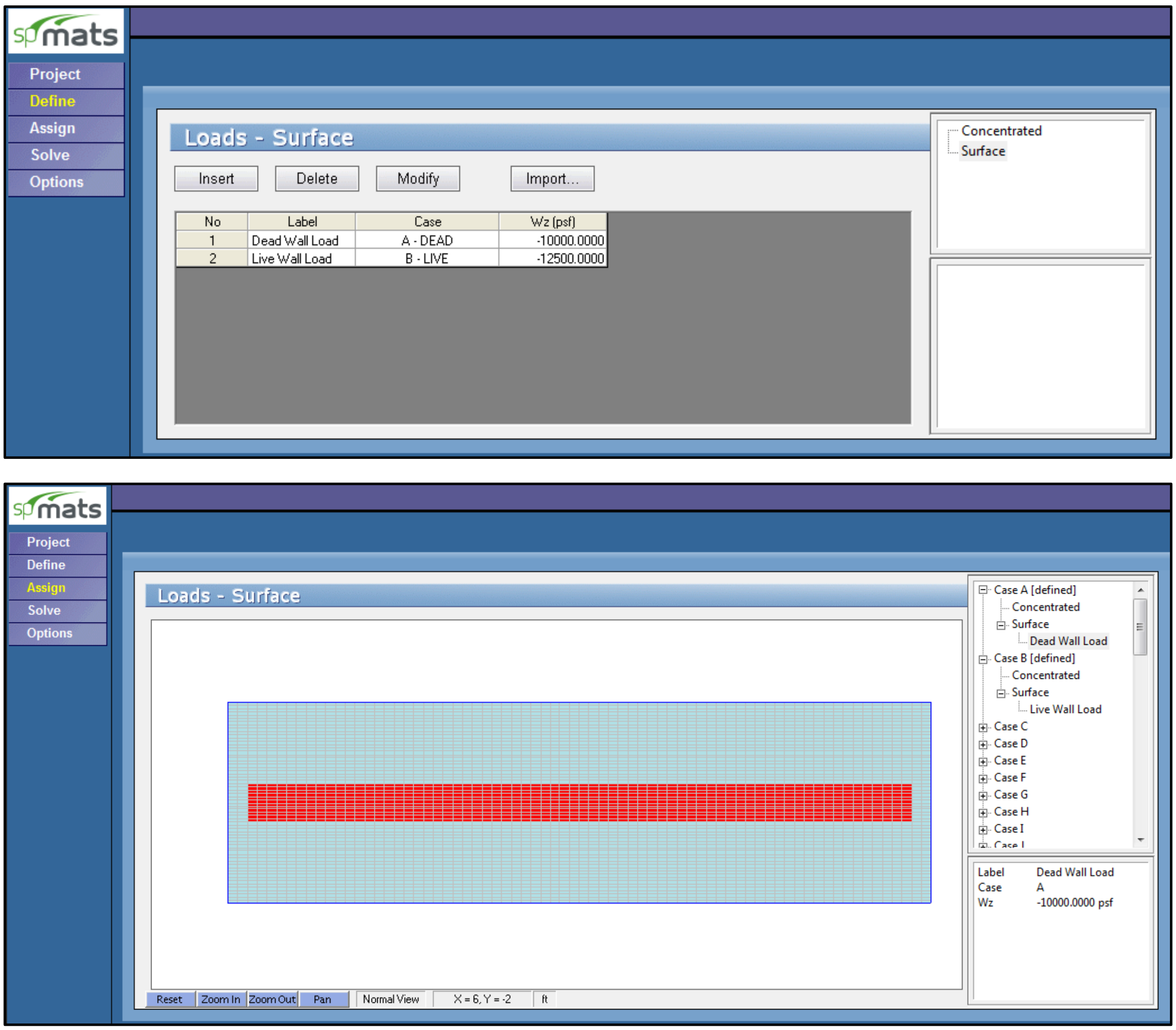


Figure 5 –Defining and Assigning Loads (spMats)

The following 3 figures provide relevant segments of spMats model results output:

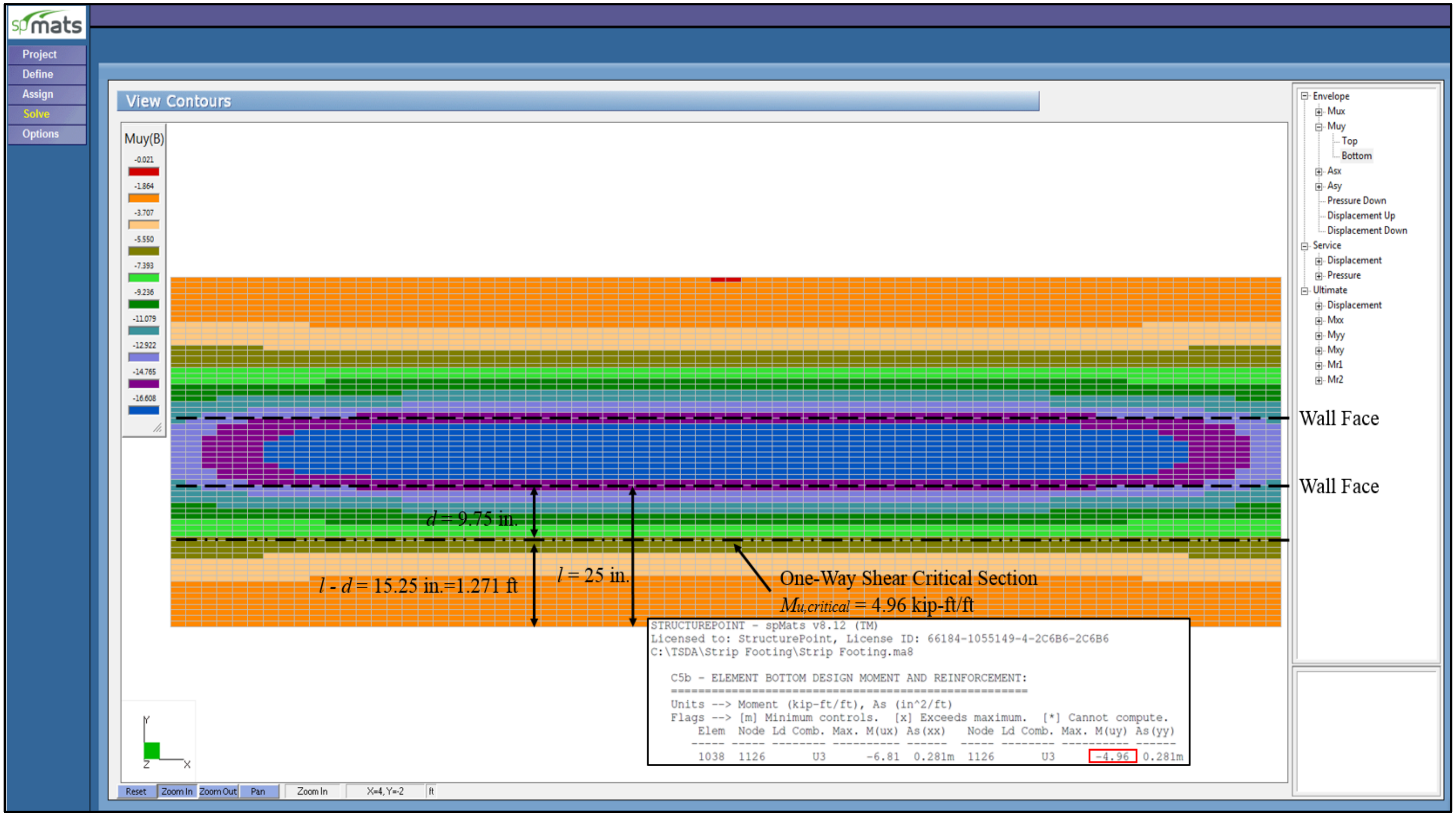


Figure 6 –Ultimate Moment Contour (spMats)

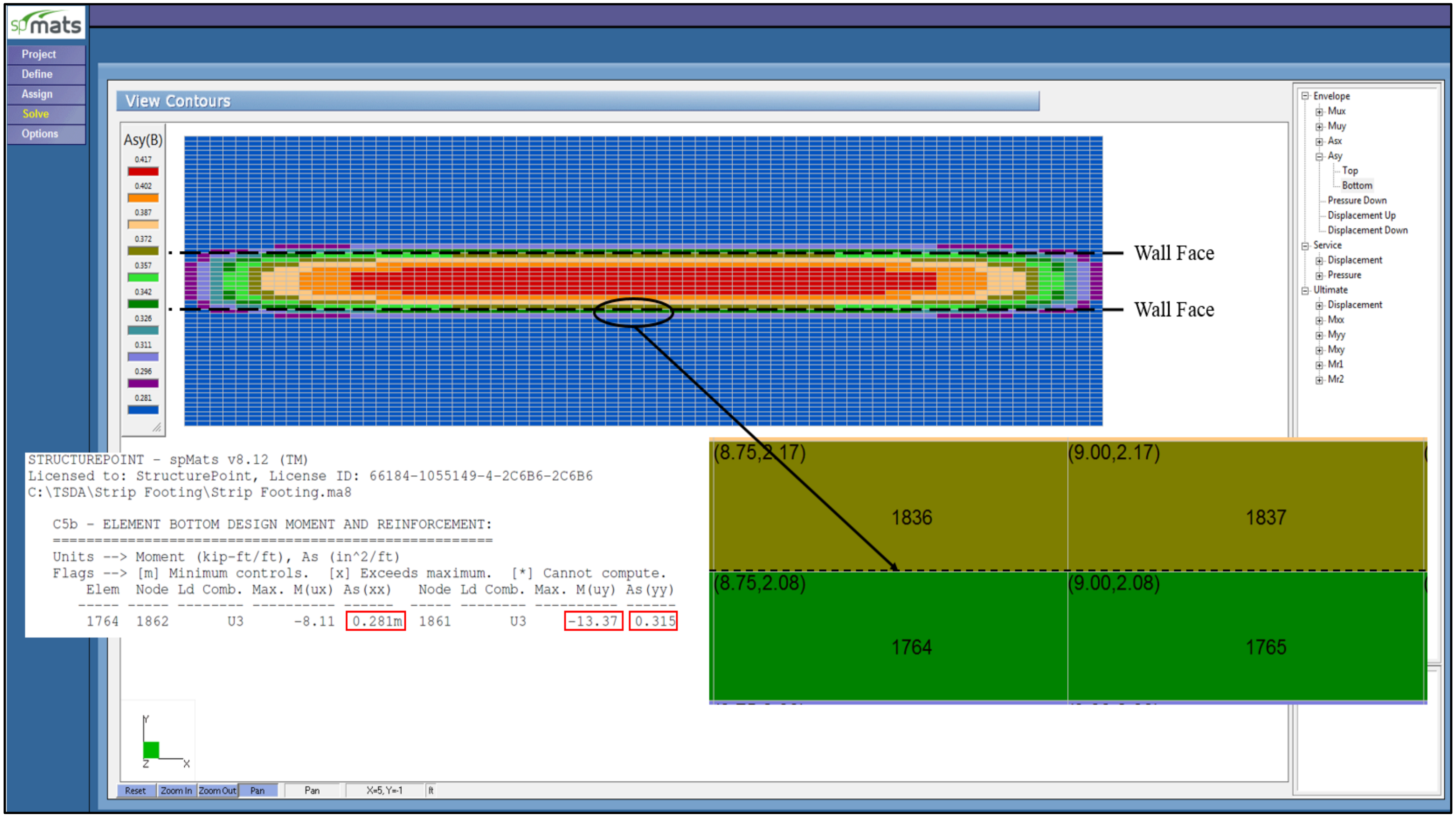


Figure 7 –Required Reinforcement Contour (spMats)

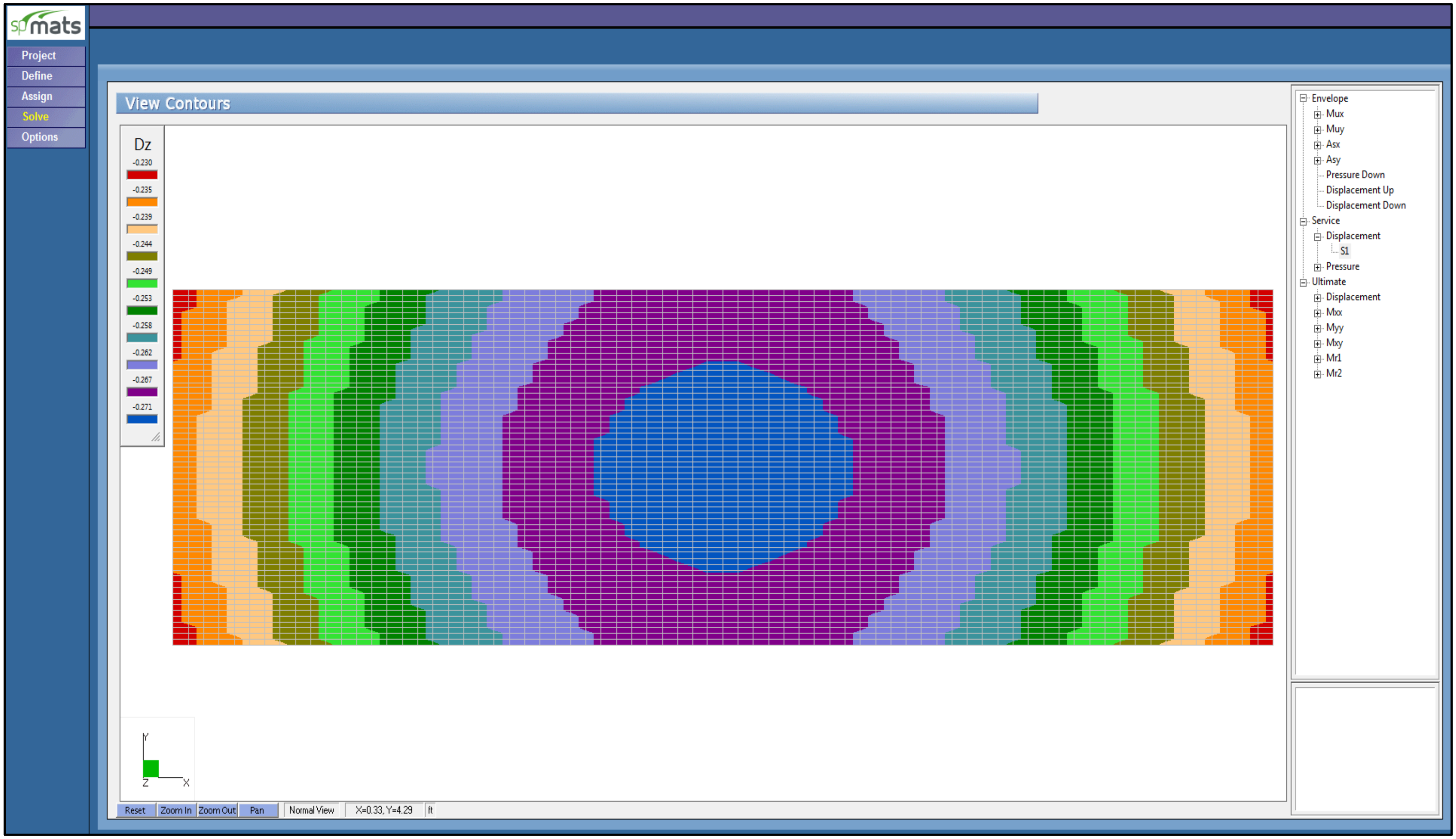


Figure 8 –Vertical Displacement Contour (spMats)

6. Design Results Comparison and Conclusions

Table 1 – Comparison of Strip Footing Analysis and Design Results				
	M_u (kip-ft/ft)	V_u (kips/ft)	$A_{s,min}$ (in.^{2}/ft)	$A_{s,required}$ (in.^{2}/ft)
Hand	13.40	7.87	0.28	0.316
Reference	13.40	8.51	0.28	0.330
spMats	13.37	7.81	0.28	0.315

Where V_u is obtained from spMats using the value of M_u at the one-way shear critical section (at distance d from the face of the wall) as follows (see Figure 6):

$$M_u = \text{ area under shear diagram} = \frac{V_u \times (l-d)}{2} \rightarrow V_u = \frac{M_u \times 2}{(l-d)} = \frac{4.96 \times 2}{1.271} = 7.81 \text{ kips/ft}$$

The results of all the hand calculations and the reference used illustrated above are in precise agreement with the automated exact results obtained from the spMats program except where the author made simplifying assumptions.

For example, the reference calculated required reinforcement area and one-way shear at the critical section based on three simplification assumptions:

1. $jd \simeq 0.95 \times d \simeq 9.025$ in. (Actual $jd = d - a/2 = 9.436$ in.)
2. $d \simeq 9.5$ in. (Actual $d = 9.75$ in. based on the bar size used)
3. $A_{tributary}$ for one-way shear in the reference example was calculated based on the initial assumption of the footing thickness (based on 12 in. footing thickness).

spMats results show exact values for *jd* and *d*, resulting in lower required area of steel. Similar differences in one-way shear values as the reference uses the initial assumption of the footing thickness to calculate the applied factored shear instead of the final selected footing thickness (13 in.) used in spMats and hand solution.

The required reinforcement is calculated in spMats by default based on maximum moment within an element (the upper left or right nodes from element 1764 as shown in Figure 7). If the “average moment within an element” option is selected by the user to compute the required reinforcement, then the averaged required reinforcement for the two adjacent elements 1764 and 1836 should be used for comparison.

When defining the design parameters in spMats, close attention should be paid to locating the top and bottom layers of reinforcement in both the x- and y-directions. In this example, the main reinforcement is located along the y-axis, then locating the reinforcement layer along the y-axis at the bottom of the reinforcement layer along the x-axis will lead to a more economical design. The following Figure shows the Design Parameters module in spMats with values used in this example where 3 in. clear cover and #4 bars are used.

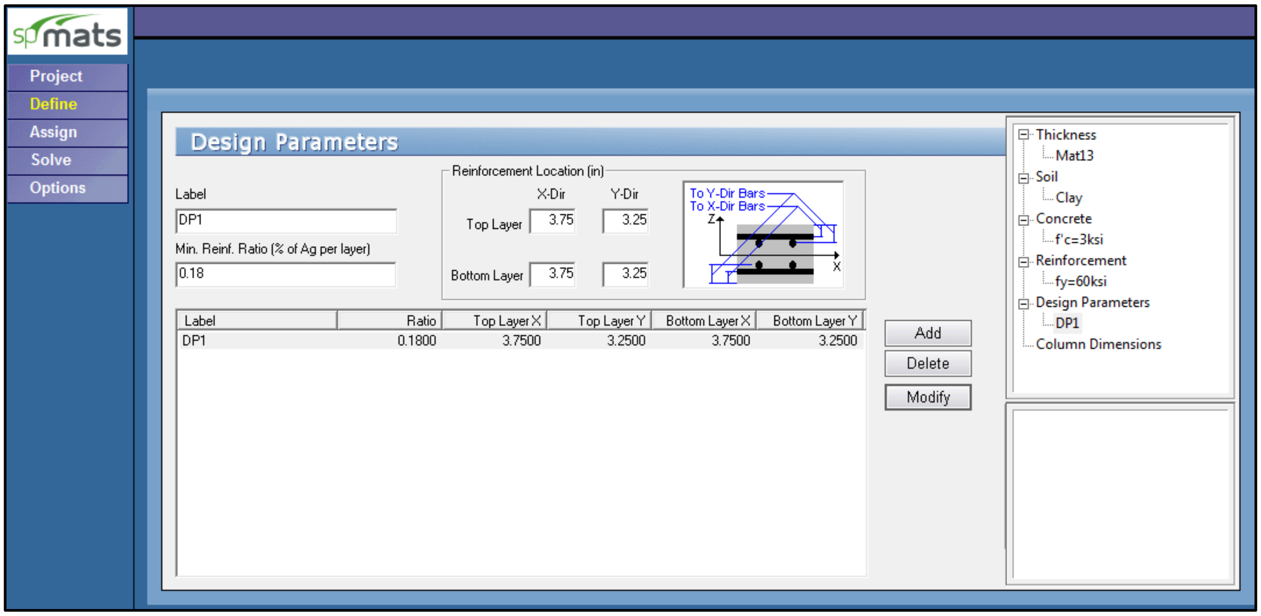


Figure 9 –Defining Design Parameters (spMats)