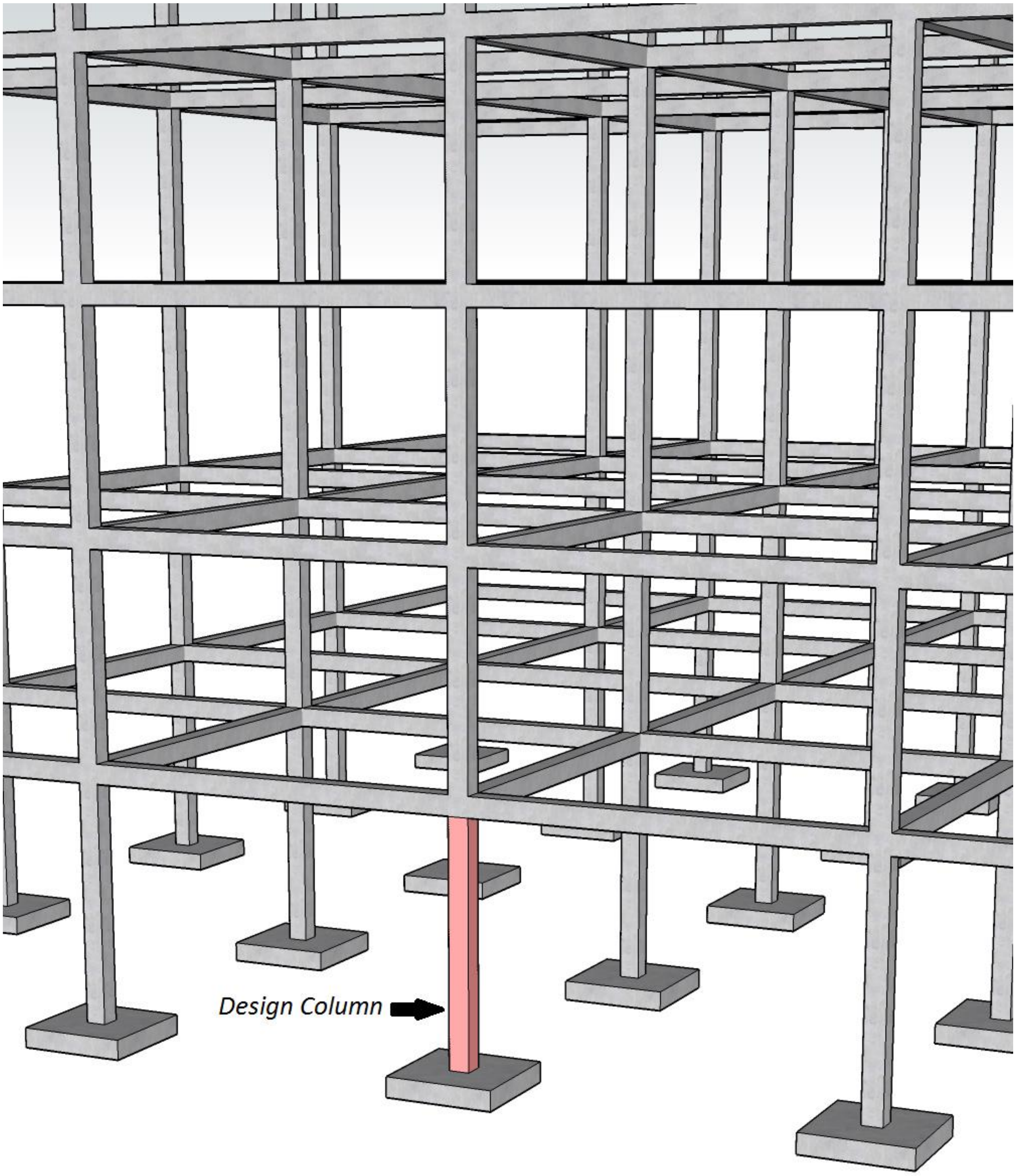
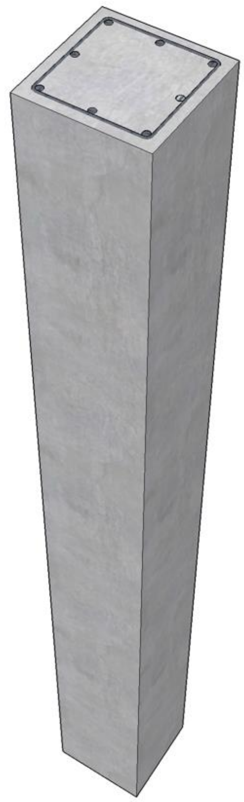
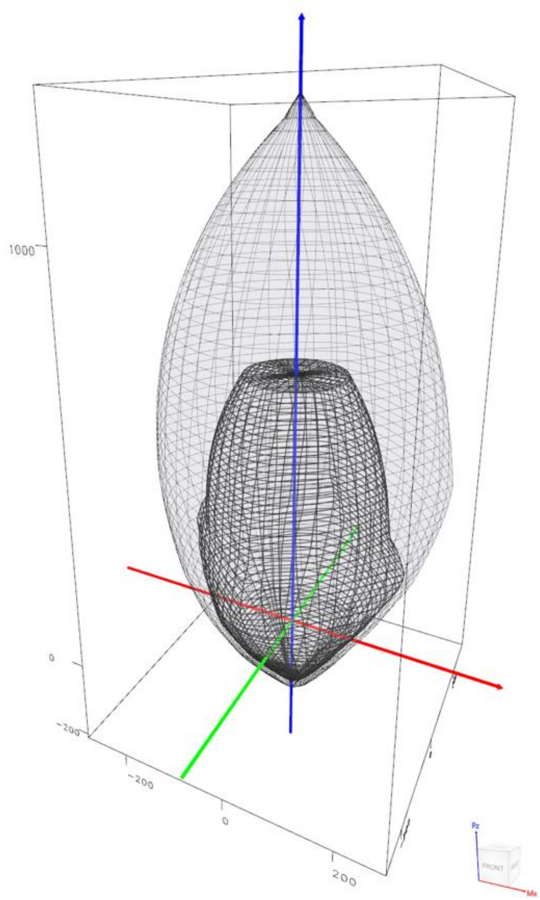
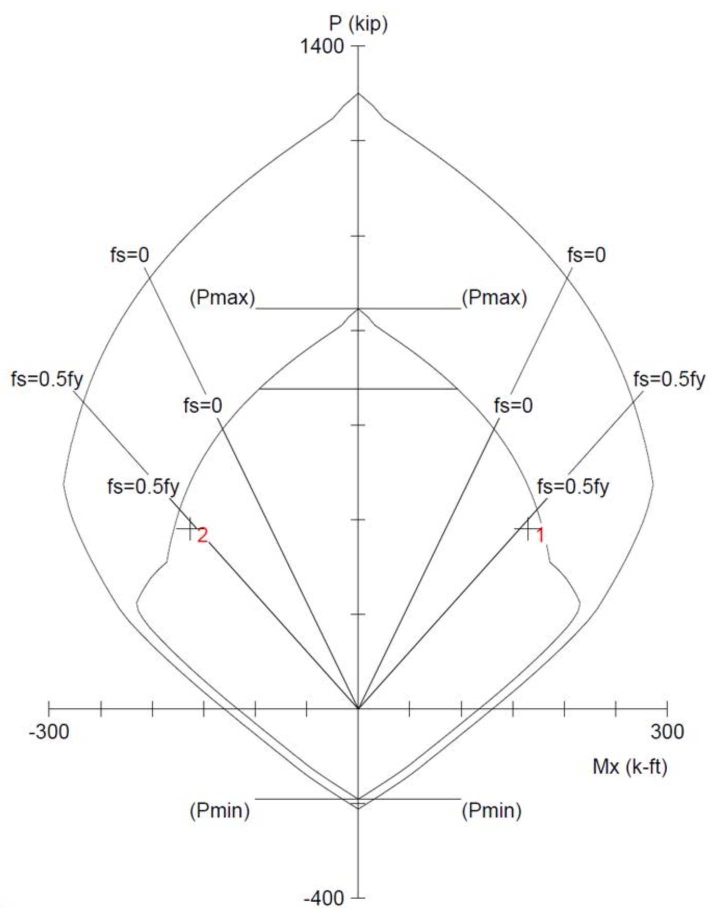
**Slenderness Effects for Concrete Columns in Sway Frame - Moment Magnification Method**






1000
0
-200
-200
0
200


P (kip)
1400
fs=0
fs=0
(Pmax)
(Pmax)
fs=0.5fy
fs=0.5fy
fs=0
fs=0
fs=0.5fy
fs=0.5fy
2
1
-300
300
Mx (k-ft)
(Pmin)
(Pmin)
-400

**Slender Concrete Column Design in Sway Frame Buildings**

Evaluate slenderness effect for columns in a sway frame multistory reinforced concrete building by designing the first story exterior column. The clear height of the first story is 15 ft-6 in., and is 9 ft. for all of the other stories. Lateral load effects on the building are governed by wind forces. Compare the calculated results with the values presented in the Reference and with exact values from spColumn engineering software program from StructurePoint.

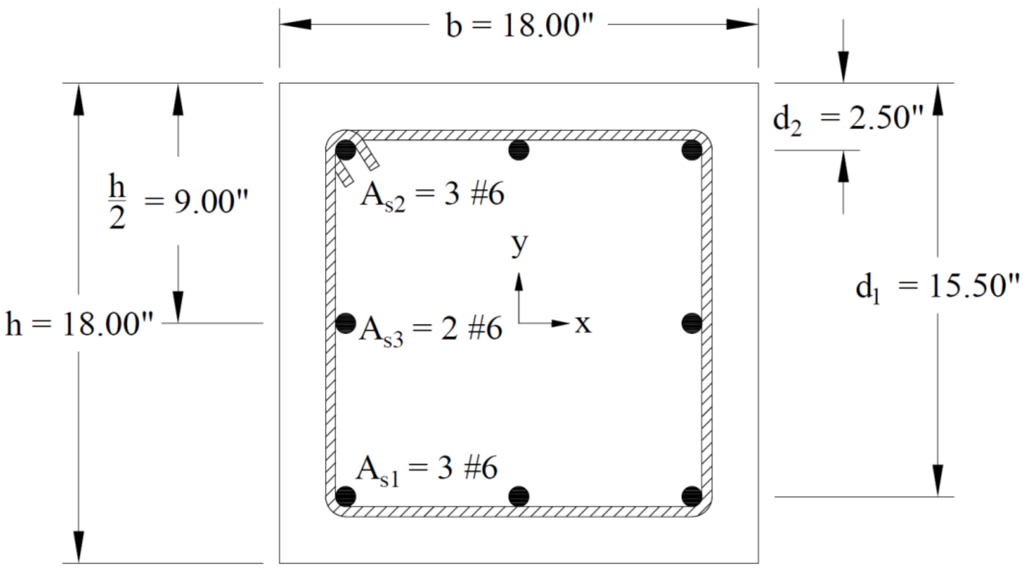



Figure 1 – Slender Reinforced Concrete Column Cross-Section

# Contents

## Code

Building Code Requirements for Structural Concrete (ACI 318-14) and Commentary (ACI 318R-14)

## Reference

Reinforced Concrete Mechanics and Design, 7th Edition, 2016, James Wight, Pearson, Example 12-3

## Design Data

$f_c'$ = 4,000 psi for columns

$f_y$ = 60,000 psi

Slab thickness = 6 in.

Exterior Columns = 18 in. x 18 in.

Interior Columns = 18 in. x 18 in.

Interior Beams = 18 in. x 30 in. x 30 ft

Exterior Beams = 18 in. x 30 in. x 32 ft

Floor superimposed dead load = 20 psf

Floor live load = 80 psf

Roof superimposed dead load = 25 psf

Roof live load = 30 psf

Wind loads computed according to ASCE 7-10

Total building loads in the first story from the reference:

Table 1 – Total building factored loads

| ASCE 7-10 Reference | No. | Load Combination | $\sum P_u$, kip |
|---|---|---|---|
| 2.3.2-1 | 1 | 1.4D | 10,990 |
| 2.3.2-2 | 2 | 1.2D + 1.6L + 0.5$L_r$ | 11,400 |
| 2.3.2-3 | 3 | 1.2D + 0.5L + 1.6 $L_r$ | 10,459 |
| | 4 | 1.2D + 1.6$L_r$ + 0.8W | 9,882 |
| | 5 | 1.2D + 1.6$L_r$ - 0.8W | 9,882 |
| 2.3.2-4 | 6 | 1.2D + 0.5L + 0.5$L_r$ + 1.6W | 10,100 |
| | 7 | 1.2D + 0.5L + 0.5$L_r$ - 1.6W | 10,100 |
| 2.3.2-6 | 8 | 0.9D + 1.6W | 7,065 |
| | 9 | 0.9D - 1.6W | 7,065 |

## 1. Factored Axial Loads and Bending Moments

### 1.1. Service loads

| Table 2 - Exterior column service loads | | | |
|---|---|---|---|
| Load Case | Axial Load, kip | Bending Moment, ft-kip | |
| | | Top | Bottom |
| Dead, D | 283.0 | -34.9 | -36.8 |
| Live, L | 42.9 | -11.2 | -11.8 |
| Roof Live, $L_r$ | 10.1 | 0.0 | 0.0 |
| Wind, W (N-S) | 9.0 | -47.8 | -46.1 |

### 1.2. Load Combinations – Factored Loads

***ASCE 7-10 (2.3.2)***

| Table 3 - Exterior column factored loads | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ASCE 7-10 Reference | No. | Load Combination | Axial Load, kip | Bending Moment, ft-kip | | $M_{Top,ns}$ ft-kip | $M_{Bottom,ns}$ ft-kip | $M_{Top,s}$ ft-kip | $M_{Bottom,s}$ ft-kip |
| | | | | Top | Bottom | | | | |
| 2.3.2-1 | 1 | 1.4D | 396.2 | 48.9 | 51.5 | 48.9 | 51.5 | --- | --- |
| 2.3.2-2 | 2 | 1.2D + 1.6L + 0.5$L_r$ | 413.3 | 59.8 | 63 | 59.8 | 63 | --- | --- |
| 2.3.2-3 | 3 | 1.2D + 0.5L + 1.6 $L_r$ | 377.2 | 47.5 | 50.1 | 47.5 | 50.1 | --- | --- |
| | 4 | 1.2D + 1.6$L_r$ + 0.8W | 363.0 | 80.1 | 81 | 41.9 | 44.2 | 38.2 | 36.9 |
| | 5 | 1.2D + 1.6$L_r$ - 0.8W | 348.6 | 3.6 | 7.3 | 41.9 | 44.2 | -38.2 | -36.9 |
| 2.3.2-4 | 6 | 1.2D + 0.5L + 0.5$L_r$ + 1.6W | 380.5 | 124.0 | 123.8 | 47.5 | 50.1 | 76.5 | 73.8 |
| | 7 | 1.2D + 0.5L + 0.5$L_r$ - 1.6W | 351.7 | -29.0 | -23.7 | 47.5 | 50.1 | -76.5 | -73.8 |
| 2.3.2-6 | 8 | 0.9D + 1.6W | 269.1 | 107.9 | 106.9 | 31.4 | 33.1 | 76.5 | 73.8 |
| | 9 | 0.9D - 1.6W | 240.3 | -45.1 | -40.6 | 31.4 | 33.1 | -76.5 | -73.8 |

## 2. Slenderness Effects and Sway or Nonsway Frame Designation

Columns and stories in structures are considered as nonsway frames if the increase in column end moments due to second-order effects does not exceed 5% of the first-order end moments, or the stability index for the story ($Q$) does not exceed 0.05. ***ACI 318-14 (6.6.4.3)***

$\sum P_u$ is the total vertical load in the first story corresponding to the lateral loading case for which $\sum P_u$ is greatest (without the wind loads, which would cause compression in some columns and tension in others and thus would cancel out). ***ACI 318-14 (6.6.4.4.1 and R6.6.4.3)***

$V_{us}$ is the factored horizontal story shear in the first story corresponding to the wind loads, and $\Delta_o$ is the first-order relative deflection between the top and bottom of the first story due to $V_{us}$. ***ACI 318-14 (6.6.4.4.1 and R6.6.4.3)***

From Table 1, load combination (2.3.2-2 No. 2) provides the greatest value of $\sum P_u$.

$$\Sigma P_u = 1.2\times D + 1.6\times L + 0.5\times L_r = 11,400 \text{ kip}$$ ***ASCE 7-10 (2.3.2-2)***

Since there is no lateral load in this load combination, the reference applied an arbitrary lateral load representing ($V_{us}$) at the top of the first story and calculated the resulting story lateral deflection ($\Delta_o$).

$$V_{us} = 20 \text{ kip (given)}$$

$$\Delta_o = 0.079 \text{ in. (given)}$$

$$Q = \frac{\Sigma P_u \times \Delta_o}{V_{us}\times l_c} = \frac{11,400\times 0.079}{20\times(18\times 12 - 30)} = 0.21 > 0.05$$ ***ACI 318-14 (Eq. 6.6.4.4.1)***

Thus, the frame at the first story level is considered sway.

### 3. Determine Slenderness Effects

$$I_{column} = 0.7 \times \frac{c^4}{12} = 0.7 \times \frac{18^4}{12} = 6{,}124 \text{ in.}^4$$ ***ACI 318-14 (Table 6.6.3.1.1(a))***

$$E_c = 57{,}000 \times \sqrt{f_c^{'}} = 57{,}000 \times \sqrt{4{,}000} = 3{,}605 \text{ ksi}$$ ***ACI 318-14 (19.2.2.1.b)***

For the column below level 2:

$$\frac{E_c \times I_{column}}{l_c} = \frac{3{,}605 \times 6{,}124}{18} = 102 \times 10^3 \text{ in.kip}$$

For the column above level 2:

$$\frac{E_c \times I_{column}}{l_c} = \frac{3{,}605 \times 6{,}124}{11.5} = 160 \times 10^3 \text{ in.kip}$$

For beams framing into the columns:

$$\frac{E_b \times I_{beam}}{l_b} = \frac{3{,}605 \times 14{,}175}{32 \times 12} = 133 \times 10^3 \text{ in.kip}$$

Where:

$$E_b = 57{,}000 \times \sqrt{f_c^{'}} = 57{,}000 \times \sqrt{4000} = 3{,}605 \text{ ksi}$$ ***ACI 318-14 (19.2.2.1.b)***

$$I_{beam} = 0.35 \times \frac{b \times h^3}{12} = 0.35 \times \frac{18 \times 30^3}{12} = 14{,}175 \text{ in.}^4$$ ***ACI 318-14 (Table 6.6.3.1.1(a))***

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{102 + 160}{133} = 1.97$$ ***ACI 318-14 (Figure R6.2.5)***

$\Psi_B = 1.0$ (Column considered fixed at the base) ***ACI 318-14 (Figure R6.2.5)***

Using Figure R6.2.5 from ACI 318-14 → $k$ = 1.44 as shown in the figure below for the exterior column.

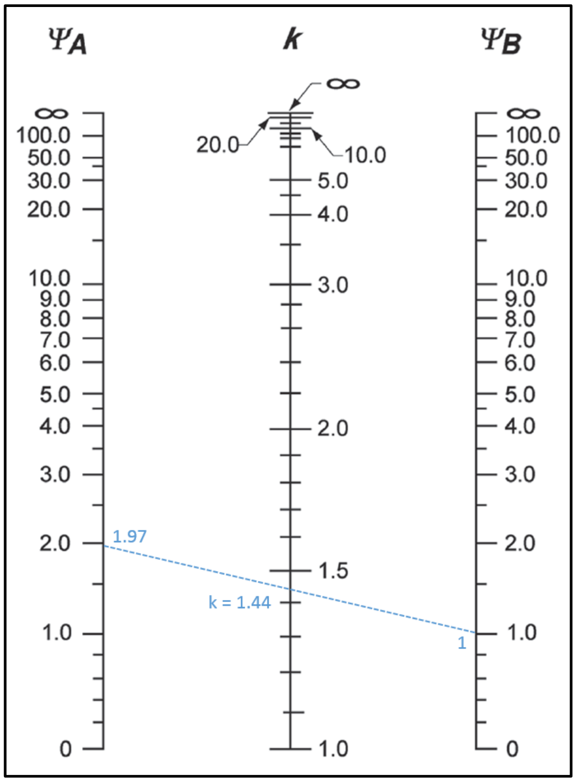



Figure 2 – Effective Length Factor ($k$) for Exterior Column (Sway Frame)

$$\frac{k \times l_u}{r} = \frac{1.44 \times 15.5}{5.196} = 51.55 > 22 \;\rightarrow \text{Consider Slenderness}$$ ***ACI 318-14 (6.2.5a)***

Where:

$r$ = radius of gyration = $(a)\ \sqrt{\dfrac{I_g}{A_g}}$ *or* (b) $0.3 \times c_1$ ***ACI 318-14 (6.2.5.1)***

$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{18^4/12}{18^2}} = 5.196 \text{ in.}$$

**4. Moment Magnification at Ends of Compression Member**

A detailed calculation for load combinations 2 and 6 is shown below to illustrate the slender column moment magnification procedure. Table 4 summarizes the magnified moment computations for the exterior columns.

4.1. Gravity Load Combination #2 (Gravity Loads Only)

$$M_2 = M_{2ns} + \delta_s M_{2s}$$ ***ACI 318-14 (6.6.4.6.1b)***

Where:

$$M_{Top\_s} = M_{Bottom\_s} = M_{2\_s} = 0 \text{ ft.kip}$$

$$\therefore M_2 = M_{2ns}$$

$$M_{Top\_2^{nd}} = M_{Top,ns} = 59.8 \text{ ft.kip}$$

$$M_{Bottom\_2^{nd}} = M_{Bottom,ns} = 63 \text{ ft.kip}$$

$$M_{2\_2^{nd}} = max\left(M_{Top\_2^{nd}}, M_{Bottom\_2^{nd}}\right) = M_{Bottom\_2^{nd}} = 63 \text{ ft.kip} \rightarrow M_{2\_1^{st}} = M_{Bottom\_1^{st}} = 63 \text{ ft.kip}$$

$$M_{1\_2^{nd}} = min\left(M_{Top\_2^{nd}}, M_{Bottom\_2^{nd}}\right) = M_{Top\_2^{nd}} = 59.8 \text{ ft.kip} \rightarrow M_{1\_1^{st}} = M_{Top\_1^{st}} = 59.8 \text{ ft.kip}$$

$P_u$ = 413.3kip

4.2. Lateral Load Combination #6 (Gravity Plus Wind Loads)

$$M_2 = M_{2ns} + \delta_s M_{2s}$$ ***ACI 318-14 (6.6.4.6.1b)***

Where:

$$\delta_s = \text{ moment magnifier} = \begin{Bmatrix} \text{(a)} \ \dfrac{1}{1-Q} \\ \text{(b)} \ \dfrac{1}{1-\dfrac{\Sigma P_u}{0.75\Sigma P_c}} \\ \text{(c) Second-order elastic analysis} \end{Bmatrix}$$ ***ACI 318-14 (6.6.4.6.2)***

There are three options for calculating $\delta_s$. ACI 318-14 (6.6.4.6.2(b)) will be used since it does not require a detailed structural analysis model results to proceed and is also used by the solver engine in spColumn.

$\sum P_u$ is the summation of all the factored vertical loads in the first story, and $\sum P_c$ is the summation of the critical buckling load for all sway-resisting columns in the first story.

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ ***ACI 318-14 (6.6.4.4.2)***

Where:

$$(EI)_{eff} = \begin{Bmatrix} \text{(a)} \ \dfrac{0.4E_cI_g}{1+\beta_{ds}} \\ \text{(b)} \ \dfrac{0.2E_cI_g + E_sI_{se}}{1+\beta_{ds}} \\ \text{(c)} \ \dfrac{E_cI}{1+\beta_{ds}} \end{Bmatrix}$$ ***ACI 318-14 (6.6.4.4.4)***

There are three options for calculating the effective flexural stiffness of slender concrete columns $(EI)_{eff}$. The second equation provides accurate representation of the reinforcement in the section and will be used in this example and is also used by the solver in spColumn. Further comparison of the available options is provided in "Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns" technical note.

$$I_{column} = \frac{c^4}{12} = \frac{18^4}{12} = 8,748 \text{ in.}^4$$ ***ACI 318-14 (Table 6.6.3.1.1(a))***

$$E_c = 57,000 \times \sqrt{f_c^{'}} = 57,000 \times \sqrt{4000} = 3,605 \text{ ksi}$$ ***ACI 318-14 (19.2.2.1.a)***

$\beta_{ds}$ is the ratio of maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination. The maximum factored sustained shear in this example is equal to zero leading to $\beta_{ds} = 0$. ***ACI 318-14 (6.6.3.1.1)***

For exterior columns with one beam framing into them in the direction of analysis (8 columns):

With 8-#6 reinforcement equally distributed on all sides $I_{se}$ = 111.5 in.$^4$ (Ref. uses approximate value of 150 in.$^4$ in lieu of exact value calculated by spColumn).

$$(EI)_{eff} = \frac{0.2E_c I_g + E_s I_{se}}{1+\beta_{ds}}$$ ***ACI 318-14 (6.6.4.4.4(b))***

$$(EI)_{eff} = \frac{0.2 \times 3,605 \times 8,748 + 29,000 \times 111.5}{1+0} = 9.5 \times 10^6 \text{ kip-in.}^2$$

$k$ = 1.44 (calculated previously).

$$P_{c1} = \frac{\pi^2 \times 9.5 \times 10^6}{(1.44 \times 15.5 \times 12)^2} = 1,313 \text{ kip}$$

For exterior columns with two beams framing into them in the direction of analysis (8 columns):

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{102+160}{133+142} = 0.95$$ ***ACI 318-14 (Figure R6.2.5)***

$$\Psi_B = 1.0 \text{ (Column essentially fixed at base)}$$ ***ACI 318-14 (Figure R6.2.5)***

Using Figure R6.2.5 from ACI 318-14 → $k$ = 1.31 as shown in the figure below for the exterior columns with two beams framing into them in the directions of analysis.

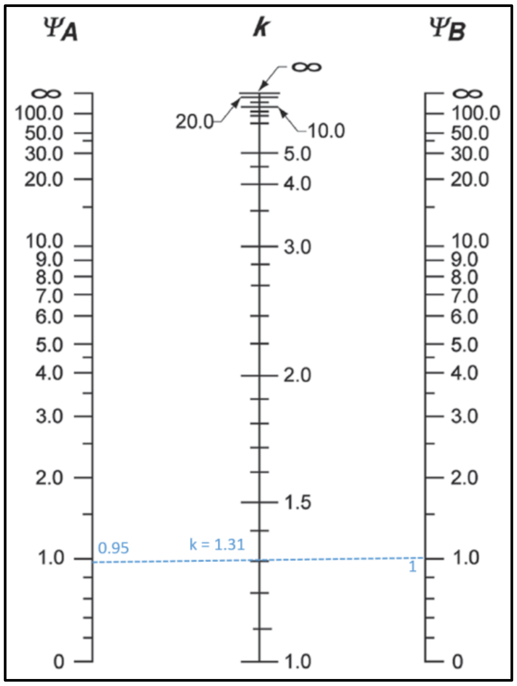



Figure 3 – Effective Length Factor ($k$) for Exterior Columns with Two Beams Framing into them in the Direction of Analysis

$$P_{c2} = \frac{\pi^2 \times 9.5 \times 10^6}{(1.31 \times 15.5 \times 12)^2} = 1{,}586 \text{ kip}$$

For interior columns (8 columns):

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{102 + 160}{133 + 142} = 0.95$$ ***ACI 318-14 (Figure R6.2.5)***

$\Psi_B = 1.0$ (Column essentially fixed at base) ***ACI 318-14 (Figure R6.2.5)***

Using Figure R6.2.5 from ACI 318-14 → $k$ = 1.31 as shown in the figure below for the interior columns.

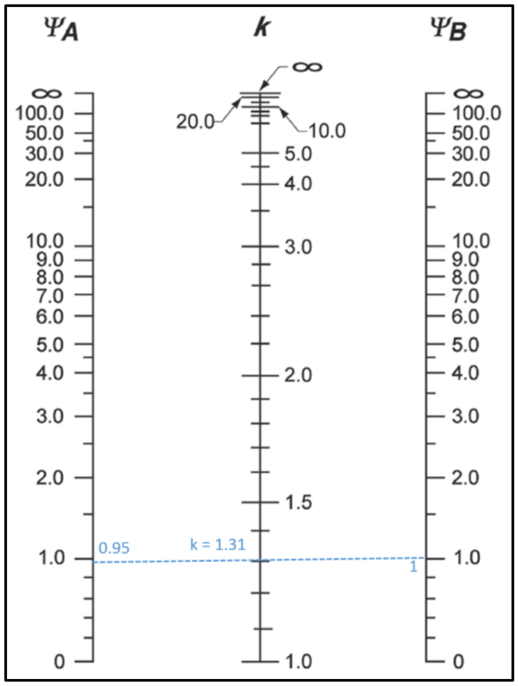



Figure 4 – Effective Length Factor ($k$) Calculations for Interior Columns

With 8-#8 reinforcement equally distributed on all sides $I_{se}$ = 192.6 in.$^4$

$$\left(EI\right)_{eff}=\frac{0.2E_cI_g+E_sI_{se}}{1+\beta_{ds}}$$ ***ACI 318-14 (6.6.4.4.4(b))***

$$\left(EI\right)_{eff}=\frac{0.2\times3,605\times6,124+29,000\times192.6}{1+0}=11.9\times10^6\text{ kip-in.}^2$$

$$P_{c3}=\frac{\pi^2\times11.9\times10^6}{\left(1.31\times15.5\times12\right)^2}=1,977\text{ kip}$$

$$\Sigma P_c=n_1\times P_{c1}+n_2\times P_{c2}+n_3\times P_{c3}$$

$$\Sigma P_c=8\times1,313+8\times1,586+8\times1,977=39,005\text{ kip}$$

$$\Sigma P_u=10,100\text{ kip (Table 1)}$$

$$\delta_s=\frac{1}{1-\dfrac{\Sigma P_u}{0.75\times\Sigma P_c}}$$ ***ACI 318-14 (6.6.4.6.2(b))***

$$\delta_s=\frac{1}{1-\dfrac{10,100}{0.75\times39,005}}=1.53$$

$$\delta_sM_{Top,s}=1.53\times76.5=117.1\text{ ft.kip}$$

$$M_{Top\_2^{nd}} = M_{Top,ns} + \delta_s M_{Top,s} = 47.5 + 117.1 = 164.5 \text{ ft.kip}$$ ***ACI 318-14 (6.6.4.6.1)***

$$\delta_s M_{Bottom,s} = 1.53 \times 73.8 = 112.9 \text{ ft.kip}$$

$$M_{Bottom\_2^{nd}} = M_{Bottom,ns} + \delta_s M_{Bottom,s} = 50.1 + 112.9 = 163.0 \text{ ft.kip}$$ ***ACI 318-14 (6.6.4.6.1)***

$$M_{2\_2^{nd}} = max\left(M_{Top\_2^{nd}}, M_{Bottom\_2^{nd}}\right) = M_{Top\_2^{nd}} = 164.5 \text{ ft.kip} \rightarrow M_{2\_1^{st}} = M_{Top\_1^{st}} = 124.0 \text{ ft.kip}$$

$$M_{1\_2^{nd}} = min\left(M_{Top\_2^{nd}}, M_{Bottom\_2^{nd}}\right) = M_{Bottom\_2^{nd}} = 163.0 \text{ ft.kip} \rightarrow M_{1\_1^{st}} = M_{Bottom\_1^{st}} = 123.8 \text{ ft.kip}$$

$P_u$ = 380.5 kip

A summary of the moment magnification factors and magnified moments for the exterior column for all load combinations using both equation options ACI 318-14 (6.6.4.6.2(a)) and (6.6.4.6.2(b)) to calculate $\delta_s$ is provided in the table below for illustration and comparison purposes. Note: The designation of $M_1$ and $M_2$ is made based on the second-order (magnified) moments and not based on the first-order (unmagnified) moments.

Table 4 - Factored Axial loads and Magnified Moments at the Ends of Exterior Column

| No. | Load Combination | Axial Load, kip | Using ACI 6.6.4.6.2(a) | | | Using ACI 6.6.4.6.2(b) | | |
|---|---|---|---|---|---|---|---|---|
| | | | $\delta_s$ | $M_1$, ft-kip | $M_2$, ft-kip | $\delta_s$ | $M_1$, ft-kip | $M_2$, ft-kip |
| 1 | 1.4D | 396.2 | * | * | * | --- | 48.9 | 51.5 |
| 2 | 1.2D + 1.6L + 0.5$L_r$ | 413.3 | --- | 59.8 | 63 | --- | 59.8 | 63 |
| 3 | 1.2D + 0.5L + 1.6 $L_r$ | 377.2 | * | * | * | --- | 47.5 | 50.1 |
| 4 | 1.2D + 1.6$L_r$ + 0.8W | 363 | * | * | * | 1.51 | 99.6 | 99.9 |
| 5 | 1.2D + 1.6$L_r$ - 0.8W | 348.6 | * | * | * | 1.51 | -11.5 | -45.9 |
| 6 | 1.2D + 0.5L + 0.5$L_r$ + 1.6W | 380.5 | 1.14 | 134.2 | 134.7 | 1.53 | 163 | 164.5 |
| 7 | 1.2D + 0.5L + 0.5$L_r$ - 1.6W | 351.7 | * | * | * | 1.53 | -62.8 | -69.6 |
| 8 | 0.9D + 1.6W | 269.1 | * | * | * | 1.32 | 130.4 | 132.2 |
| 9 | 0.9D - 1.6W | 240.3 | * | * | * | 1.32 | -64.1 | -69.4 |

* Not covered by the reference

## 5. Moment Magnification along Length of Compression Member

In sway frames, second-order effects shall be considered along the length of columns. It shall be permitted to account for these effects using ***ACI 318-14 (6.6.4.5)*** (Nonsway frame procedure), where $C_m$ is calculated using $M_1$ and $M_2$ from ***ACI 318-14 (6.6.4.6.1)*** as follows: ***ACI 318-14 (6.6.4.6.4)***

$$M_{c2} = \delta M_2$$ ***ACI 318-14 (6.6.4.5.1)***

Where:

$M_2$ = the second-order factored moment.

$$\delta = \text{ magnification factor} = \frac{C_m}{1-\dfrac{P_u}{0.75P_c}} \geq 1.0$$ ***ACI 318-14 (6.6.4.5.2)***

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$ ***ACI 318-14 (6.6.4.4.2)***

Where:

$$(EI)_{eff} = \begin{cases} \text{(a)} & \dfrac{0.4E_cI_g}{1+\beta_{dns}} \\ \text{(b)} & \dfrac{0.2E_cI_g + E_sI_{se}}{1+\beta_{dns}} \\ \text{(c)} & \dfrac{E_cI}{1+\beta_{dns}} \end{cases}$$ ***ACI 318-14 (6.6.4.4.4)***

There are three options for calculating the effective flexural stiffness of slender concrete columns $(EI)_{eff}$. The second equation provides accurate representation of the reinforcement in the section and will be used in this example and is also used by the solver in spColumn. Further comparison of the available options is provided in "Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns" technical note.

### 5.1. Gravity Load Combination #2 (Gravity Loads Only)

$$I_{column} = \frac{c^4}{12} = \frac{18^4}{12} = 6{,}124 \text{ in.}^4$$ ***ACI 318-14 (Table 6.6.3.1.1(a))***

$$E_c = 57{,}000 \times \sqrt{f_c^{'}} = 57{,}000 \times \sqrt{4000} = 3{,}605 \text{ ksi}$$ ***ACI 318-14 (19.2.2.1.a)***

$\beta_{dns}$ is the ratio of maximum factored sustained axial load to maximum factored axial load associated with the same load combination. ***ACI 318-14 (6.6.4.4.4)***

$$P_{u,sustained} = 1.2 \times 283 = 340 \text{ kip}$$

$$P_u = 1.2 \times 283 + 1.6 \times 42.9 + 0.5 \times 10.1 = 413.3 \text{ kip}$$

$$\beta_{dns} = \frac{P_{u,sustained}}{P_u} = \frac{340}{413.3} = 0.82 < 1.00 \rightarrow \therefore \beta_{dns} = 0.82$$

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{102+160}{133} = 1.97 \text{ (Calculated previously)}$$ ***ACI 318-14 (Figure R6.2.5)***

$$\Psi_B = 1.0 \text{ (Column essentially fixed at base)}$$ ***ACI 318-14 (Figure R6.2.5)***

Using Figure R6.2.5(a) from ACI 318-14 → $k$ = 0.81 as shown in the figure below for the exterior column.

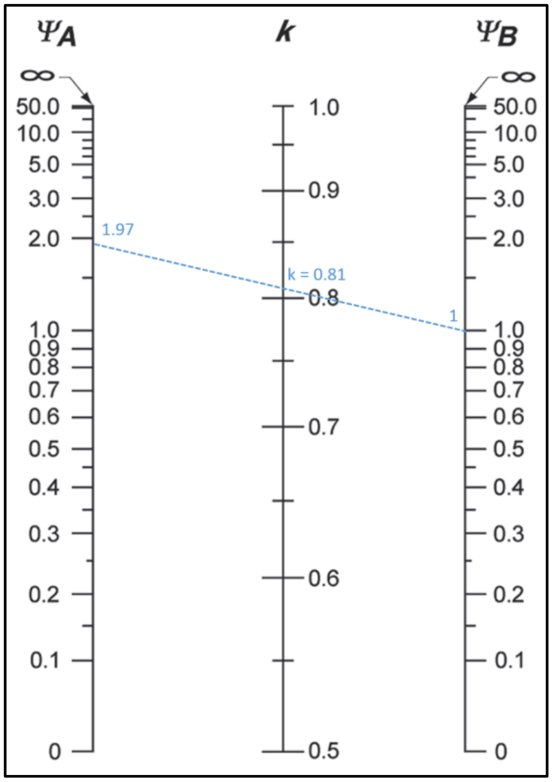



Figure 5 – Effective Length Factor ($k$) Calculations for Exterior Column (Nonsway)

With 8-#6 reinforcement equally distributed on all sides $I_{se}$ = 111.5 in.$^4$ (Ref. uses approximate value of 150 in.$^4$ in lieu of exact value calculated by spColumn).

$$(EI)_{eff} = \frac{0.2 E_c I_g + E_s I_{se}}{1+\beta_{dns}}$$ ***ACI 318-14 (6.6.4.4.4(b))***

$$(EI)_{eff} = \frac{0.2 \times 3,605 \times 6,124 + 29,000 \times 111.5}{1+0.82} = 5.2 \times 10^6 \text{ kip-in.}^2$$

$$P_c = \frac{\pi^2 \times 5.2 \times 10^6}{(0.81 \times 15.5 \times 12)^2} = 2,277 \text{ kip}$$

$$P_u = 1.2 \times 283 + 1.6 \times 42.9 + 0.5 \times 10.1 = 413.3 \text{ kip}$$ ***ASCE 7-10 (2.3.2-2)***

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2}$$ ***ACI 318-14 (6.6.4.5.3a)***

$$M_2 = M_{2\_2^{nd}} = 63.04 \text{ ft.kip (as concluded from section 4)}$$ ***ACI 318-14 (6.6.4.6.4)***

$$M_1 = M_{1\_2^{nd}} = 59.8 \text{ ft.kip (as concluded from section 4)}$$ ***ACI 318-14 (6.6.4.6.4)***

Since the column is bent in double curvature, $M_1/M_2$ is positive. ***ACI 318-14 (6.6.4.5.3)***

$$C_m = 0.6 - 0.4\left(\frac{59.8}{63.04}\right) = 0.221$$

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{0.75 P_c}} \geq 1.0$$ ***ACI 318-14 (6.6.4.5.2)***

$$\delta = \frac{0.221}{1 - \dfrac{413.3}{0.75 \times 2,277}} = 0.291 < 1.00 \rightarrow \delta = 1.00$$

$$M_{min} = P_u (0.6 + 0.03h)$$ ***ACI 318-14 (6.6.4.5.4)***

Where $P_u$ = 413.3 kip, and $h$ = the section dimension in the direction being considered = 18 in.

$$M_{min} = 413.3\left(\frac{0.6 + 0.03 \times 18}{12}\right) = 39.3 \text{ ft.kip}$$

$$M_1 = 59.8 \text{ ft.kip} > M_{min} = 39.3 \text{ ft.kip} \rightarrow M_1 = 59.8 \text{ ft.kip}$$ ***ACI 318-14 (6.6.4.5.4)***

$$M_{c1} = \delta M_1$$ ***ACI 318-14 (6.6.4.5.1)***

$$M_{c1} = 1.00 \times 59.8 = 59.8 \text{ ft.kip}$$

$$M_2 = 63.04 \text{ ft.kip} > M_{2,min} = 39.3 \text{ ft.kip} \rightarrow M_2 = 63.04 \text{ ft.kip}$$ ***ACI 318-14 (6.6.4.5.4)***

$$M_{c2} = \delta M_2$$ ***ACI 318-14 (6.6.4.5.1)***

$$M_{c2} = 1.00 \times 63.04 = 63.04 \text{ ft.kip}$$

$M_{c1}$ and $M_{c2}$ will be considered separately to ensure proper comparison of resulting magnified moments against negative and positive moment capacities of unsymmetrical sections as can be seen in the following figure.

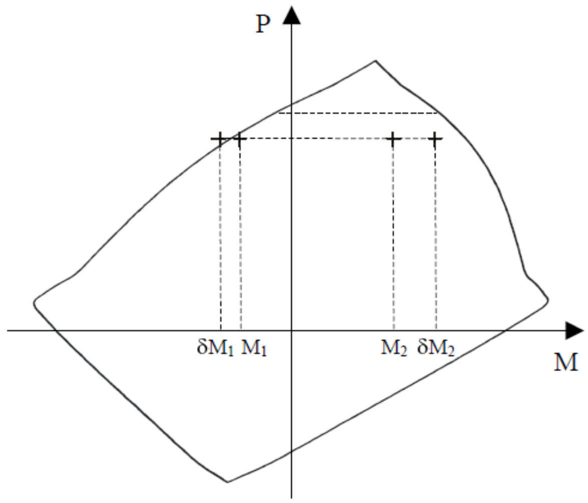



Figure 6 – Column Interaction Diagram for Unsymmetrical Section

5.2. Load Combination #6 (Gravity Plus Wind Loads)

$$I_{column} = \frac{c^4}{12} = \frac{18^4}{12} = 6{,}124 \text{ in.}^4$$ ***ACI 318-14 (Table 6.6.3.1.1(a))***

$$E_c = 57{,}000 \times \sqrt{f_c^{'}} = 57{,}000 \times \sqrt{4000} = 3{,}605 \text{ ksi}$$ ***ACI 318-14 (19.2.2.1.a)***

$\beta_{dns}$ is the ratio of maximum factored sustained axial load to maximum factored axial load associated with the same load combination. ***ACI 318-14 (6.6.4.4.4)***

$$P_{u,sustained} = 1.2 \times 283 = 340 \text{ kip}$$

$$P_u = 1.2 \times 283 + 0.5 \times 42.9 + 0.5 \times 10.1 + 1.6 \times 9 = 380.5 \text{ kip}$$

$$\beta_{dns} = \frac{P_{u,sustained}}{P_u} = \frac{340}{380.5} = 0.89 < 1.00 \rightarrow \therefore \beta_{dns} = 0.89$$

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c}\right)_{columns}}{\left(\sum \frac{EI}{l}\right)_{beams}} = \frac{102 + 160}{133} = 1.97 \text{ (Calculated previously)}$$ ***ACI 318-14 (Figure R6.2.5)***

$$\Psi_B = 1.0 \text{ (Column essentially fixed at base)}$$ ***ACI 318-14 (Figure R6.2.5)***

Using Figure R6.2.5(a) from ACI 318-14 → $k$ = 0.81

With 8-#6 reinforcement equally distributed on all sides $I_{se}$ = 111.5 in.$^4$ (Ref. uses approximate value of 150 in.$^4$ in lieu of exact value calculated by spColumn).

$$(EI)_{eff} = \frac{0.2E_cI_g + E_sI_{se}}{1+\beta_{dns}}$$ ***ACI 318-14 (6.6.4.4.4(b))***

$$(EI)_{eff} = \frac{0.2\times 3,605\times 6,124 + 29,000\times 111.5}{1+0.89} = 5.0\times 10^6 \text{ kip-in.}^2$$

$$P_c = \frac{\pi^2\times 5.0\times 10^6}{(0.81\times 15.5\times 12)^2} = 2,192 \text{ kip}$$

$$P_u = 1.2\times 283 + 0.5\times 42.9 + 0.5\times 10.1 + 1.6\times 9 = 380.5 \text{ kip}$$ ***ASCE 7-10 (2.3.2-4)***

$$C_m = 0.6 + 0.4\frac{M_1}{M_2}$$ ***ACI 318-14 (6.6.4.5.3a)***

$M_2 = M_{2\_2^{nd}} = 164.5$ ft.kip (as concluded from section 4) ***ACI 318-14 (6.6.4.6.4)***

$M_1 = M_{1\_2^{nd}} = 163.0$ ft.kip (as concluded from section 4) ***ACI 318-14 (6.6.4.6.4)***

Since the column is bent in double curvature, $M_1/M_2$ is positive. ***ACI 318-14 (6.6.4.5.3)***

$$C_m = 0.6 - 0.4\left(\frac{163.0}{164.5}\right) = 0.204$$

$$\delta = \frac{C_m}{1-\frac{P_u}{0.75P_c}} \geq 1.0$$ ***ACI 318-14 (6.6.4.5.2)***

$$\delta = \frac{0.204}{1-\frac{380.5}{0.75\times 2,192}} = 0.265 < 1.00 \rightarrow \delta = 1.00$$

$$M_{min} = P_u(0.6 + 0.03h)$$ ***ACI 318-14 (6.6.4.5.4)***

Where $P_u$ = 380.5 kip, and $h$ = the section dimension in the direction being considered = 18 in.

$$M_{min} = 380.5\left(\frac{0.6+0.03\times 18}{12}\right) = 36.1 \text{ ft.kip}$$

$$M_1 = 163.0 \text{ ft.kip} > M_{min} = 36.1 \text{ ft.kip} \rightarrow M_1 = 163.0 \text{ ft.kip}$$ ***ACI 318-14 (6.6.4.5.4)***

$$M_{c1} = \delta M_1$$ ***ACI 318-14 (6.6.4.5.1)***

$$M_{c1} = 1.00\times 163.0 = 163.0 \text{ ft.kip}$$

$$M_2 = 164.5 \text{ ft.kip} > M_{2,min} = 36.1 \text{ ft.kip} \rightarrow M_2 = 164.5 \text{ ft.kip}$$ ***ACI 318-14 (6.6.4.5.4)***

$$M_{c2} = \delta M_2$$ ***ACI 318-14 (6.6.4.5.1)***

$M_{c2} = 1.00 \times 164.5 = 164.5$ ft.kip

$M_{c1}$ and $M_{c2}$ are considered separately to ensure proper comparison of resulting magnified moments against negative and positive moment capacities of unsymmetrical sections.

A summary of the moment magnification factors and magnified moments for the exterior column for all load combinations using both equation options ACI 318-14 (6.6.4.6.2(a)) and (6.6.4.6.2(b)) to calculate $\delta_s$ is provided in the table below for illustration and comparison purposes.

| Table 5 - Factored Axial loads and Magnified Moments along Exterior Column Length | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| No. | Load Combination | Axial Load, kip | Using ACI 6.6.4.6.2(a) | | | Using ACI 6.6.4.6.2(b) | | |
| | | | δ | $M_{c1}$,ft-kip | $M_{c2}$, ft-kip | δ | $M_{c1}$, ft-kip | $M_{c2}$, ft-kip |
| 1 | 1.4D | 396.2 | * | * | * | 1 | 48.9 | 51.5 |
| 2 | 1.2D + 1.6L + 0.5$L_r$ | 413.3 | 1 | 59.8 | 63 | 1 | 59.8 | 63 |
| 3 | 1.2D + 0.5L + 1.6 $L_r$ | 377.2 | * | * | * | 1 | 47.5 | 50.1 |
| 4 | 1.2D + 1.6$L_r$ + 0.8W | 363 | * | * | * | 1 | 99.6 | 99.9 |
| 5 | 1.2D + 1.6$L_r$ - 0.8W | 348.6 | * | * | * | 1 | -33.1 | -33.1 |
| 6 | 1.2D + 0.5L + 0.5$L_r$ + 1.6W | 380.5 | 1 | 134.2 | 134.7 | 1 | 163 | 164.5 |
| 7 | 1.2D + 0.5L + 0.5$L_r$ - 1.6W | 351.7 | * | * | * | 1 | -62.8 | -69.6 |
| 8 | 0.9D + 1.6W | 269.1 | * | * | * | 1 | 130.4 | 132.2 |
| 9 | 0.9D - 1.6W | 240.3 | * | * | * | 1 | -64.1 | -69.4 |
| * Not covered by the reference | | | | | | | | |

For column design ACI 318 requires the second-order moment to first-order moment ratios should not exceed 1.40. If this value is exceeded, the column design needs to be revised. ***ACI 318-14 (6.2.6)***

| Table 6 - Second-Order Moment to First-Order Moment Ratios | | | | | |
|---|---|---|---|---|---|
| No. | Load Combination | Using ACI 6.6.4.6.2(a) | | Using ACI 6.6.4.6.2(b) | |
| | | $M_{c1}/M_{1(1st)}$ | $M_{c2}/M_{2(1st)}$ | $M_{c1}/M_{1(1st)}$ | $M_{c2}/M_{2(1st)}$ |
| 1 | 1.4D | ** | ** | 1.00* | 1.00* |
| 2 | 1.2D + 1.6L + 0.5$L_r$ | 1.00* | 1.00* | 1.00* | 1.00* |
| 3 | 1.2D + 0.5L + 1.6 $L_r$ | ** | ** | 1.00* | 1.00* |
| 4 | 1.2D + 1.6$L_r$ + 0.8W | ** | ** | 1.24 | 1.23 |
| 5 | 1.2D + 1.6$L_r$ - 0.8W | ** | ** | 1.00* | 1.00* |
| 6 | 1.2D + 0.5L + 0.5$L_r$ + 1.6W | 1.08 | 1.09 | 1.32 | 1.33 |
| 7 | 1.2D + 0.5L + 0.5$L_r$ - 1.6W | ** | ** | 1.40 < 1.88 | 1.40 < 2.08 |
| 8 | 0.9D + 1.6W | ** | ** | 1.22 | 1.23 |
| 9 | 0.9D - 1.6W | ** | ** | 1.40 < 1.58 | 1.40 < 1.54 |

* Cutoff value of $M_{min}$ is applied to $M_{1(1st)}$ and $M_{2(1st)}$ in order to avoid unduly large ratios in cases where $M_{1(1st)}$ and $M_{2(1st)}$ moments are smaller than $M_{min}$.
** Not covered by the reference

## 6. Column Design

Based on the factored axial loads and magnified moments considering slenderness effects, the capacity of the assumed column section (18 in. x 18 in. with 8-#6 bars distributed all sides equal) will be checked and confirmed to finalize the design. A column interaction diagram will be generated using strain compatibility analysis, the detailed procedure to develop column interaction diagram can be found in "Interaction Diagram – Tied Reinforced Concrete Column" example.

The axial compression capacity $\phi P_n$ for all load combinations will be set equals to $P_u$, then the moment capacity $\phi M_n$ associated to $\phi P_n$ will be compared with the magnified applied moment $M_u$. The design check for load combination #6 is shown below for illustration. The rest of the checks for the other load combinations are shown in the following Table.

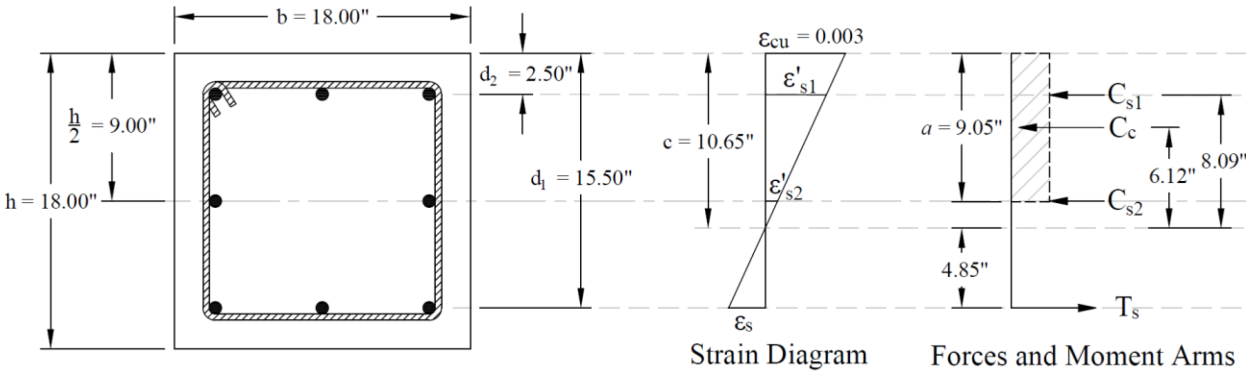



Figure 7 – Strains, Forces, and Moment Arms (Load Combination 6)

The following procedure is used to determine the nominal moment capacity by setting the design axial load capacity, $\phi P_n$, equal to the applied axial load, $P_u$ and iterating on the location of the neutral axis.

### 6.1. c, a, and strains in the reinforcement

Try $c = 10.65$ in.

Where $c$ is the distance from the fiber of maximum compressive strain to the neutral axis.

***ACI 318-14 (22.2.2.4.2)***

$$a = \beta_1 \times c = 0.85 \times 10.65 = 9.053 \text{ in.}$$ ***ACI 318-14 (22.2.2.4.1)***

Where:

$$\beta_1 = 0.85 - \frac{0.05 \times \left( f_c^{'} \times 4000 \right)}{1000} = 0.85 - \frac{0.05 \times \left( 4000 \times 4000 \right)}{1000} = 0.85$$ ***ACI 318-14 (Table 22.2.2.4.3)***

$$\varepsilon_{cu} = 0.003$$ ***ACI 318-14 (22.2.2.1)***

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60}{29,000} = 0.00207$$

$$\varepsilon_s = (d_1 - c) \times \frac{0.003}{c} = (15.50 - 10.65) \times \frac{0.003}{10.65} = 0.00137 \text{ (Tension)} < \varepsilon_y$$

$\therefore$ tension reinforcement has not yielded

$\therefore \phi = 0.65$ **_ACI 318-14 (Table 21.2.2)_**

$$\varepsilon_{s1}^{'} = (c - d_2) \times \frac{0.003}{c} = (10.65 - 2.50) \times \frac{0.003}{10.65} = 0.0023 \text{ (Compression)} > \varepsilon_y$$

$$\varepsilon_{s2}^{'} = \left(c - \frac{h}{2}\right) \times \frac{0.003}{c} = (10.65 - 9.00) \times \frac{0.003}{10.65} = 0.00046 \text{ (Tension)} < \varepsilon_y$$

6.2. Forces in the concrete and steel

$C_c = 0.85 \times f_c^{'} \times a \times b = 0.85 \times 4,000 \times 9.053 \times 18 = 554 \text{ kip}$ **_ACI 318-14 (22.2.2.4.1)_**

$$f_s = \varepsilon_s \times E_s = 0.00137 \times 29,000,000 = 39,620 \text{ psi}$$

$$\text{T}_s = f_y \times A_{s1} = 39,620 \times (3 \times 0.44) = 52.3 \text{ kip}$$

Since $\varepsilon_{s1}^{'} > \varepsilon_y \rightarrow$ compression reinforcement has yielded

$$\therefore f_{s1}^{'} = f_y = 60,000 \text{ psi}$$

Since $\varepsilon_{s2}^{'} < \varepsilon_y \rightarrow$ compression reinforcement has not yielded

$$\therefore f_{s2}^{'} = \varepsilon_{s2}^{'} \times E_s = 0.00046 \times 29,000,000 = 13,479 \text{ psi}$$

The area of the reinforcement in this layer has been included in the area ($ab$) used to compute $C_c$. As a result, it is necessary to subtract 0.85$f_c$' from $f_s$' before computing $C_s$:

$$\text{C}_{s1} = \left(f_{s1}^{'} - 0.85 f_c^{'}\right) \times A_{s1}^{'} = (60,000 - 0.85 \times 4,000) \times (3 \times 0.44) = 74.7 \text{ kip}$$

$$\text{C}_{s2} = \left(f_{s2}^{'} - 0.85 f_c^{'}\right) \times A_{s2}^{'} = (13,479 - 0.85 \times 4,000) \times (2 \times 0.44) = 8.9 \text{ kip}$$

6.3. $\phi P_n$ and $\phi M_n$

$$P_n = C_c + C_{s1} + C_{s2} - T_s = 554.0 + 74.7 + 8.9 - 52.3 = 585 \text{ kip}$$

$$\phi P_n = 0.85 \times 585 = 380.4 \text{ kip} = P_u$$

The assumed value of c = 10.65 in. is correct.

$$M_n = C_c \times \left(\frac{h}{2} - \frac{a}{2}\right) + C_{s1} \times \left(\frac{h}{2} - d_2\right) + C_{s2} \times \left(\frac{h}{2} - \frac{h}{2}\right) + T_s \times \left(d_1 - \frac{h}{2}\right)$$

$$M_n = 554.0 \times \left(\frac{18}{2} - \frac{9.053}{2}\right) + 74.7 \times \left(\frac{18}{2} - 2.5\right) + 8.9 \times \left(\frac{18}{2} - \frac{18}{2}\right) + 52.3 \times \left(10.65 - \frac{18}{2}\right) = 275\,\text{kip.ft}$$

$$\phi M_n = 0.85 \times 275 = 178.97\,\text{kip.ft} > M_u = M_{c2} = 164.5\,\text{kip.ft}$$

Table 7 – Exterior Column Axial and Moment Capacities

| No. | $P_u$, kip | $M_u = M_{2(2nd)}$, ft-kip | c, in. | $\varepsilon_t = \varepsilon_s$ | φ | $\varphi P_n$, kip | $\varphi M_n$, kip.ft |
|---|---|---|---|---|---|---|---|
| 1 | 396.2 | 51.5 | 10.98 | 0.00123 | 0.65 | 396.2 | 177.0 |
| 2 | 413.3 | 63.0 | 11.35 | 0.00110 | 0.65 | 413.3 | 174.7 |
| 3 | 377.2 | 50.1 | 10.55 | 0.00141 | 0.65 | 377.6 | 179.5 |
| 4 | 363.0 | 99.9 | 10.25 | 0.00154 | 0.65 | 363.0 | 181.1 |
| 5 | 348.6 | -33.1 | 9.96 | 0.00167 | 0.65 | 348.6 | 182.6 |
| 6 | 380.5 | 164.5 | 10.65 | 0.00137 | 0.65 | 380.4 | 179.0 |
| 7 | 351.7 | -69.6 | 10.02 | 0.00164 | 0.65 | 351.6 | 182.3 |
| 8 | 269.1 | 132.2 | 7.26 | 0.00340 | 0.77 | 269.2 | 205.1 |
| 9 | 240.3 | -69.4 | 6.36 | 0.00431 | 0.84 | 240.2 | 212.2 |

Since $\phi M_n > M_u$ for all $\phi P_n = P_u$, use 18 x 18 in. column with 8-#6 bars.

## 7. Column Interaction Diagram - spColumn Software

spColumn program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames. For this column section, we ran in investigation mode with control points using the 318-14. In lieu of using program shortcuts, spSection (Figure 8) was used to place the reinforcement and define the cover to illustrate handling of irregular shapes and unusual bar arrangement.

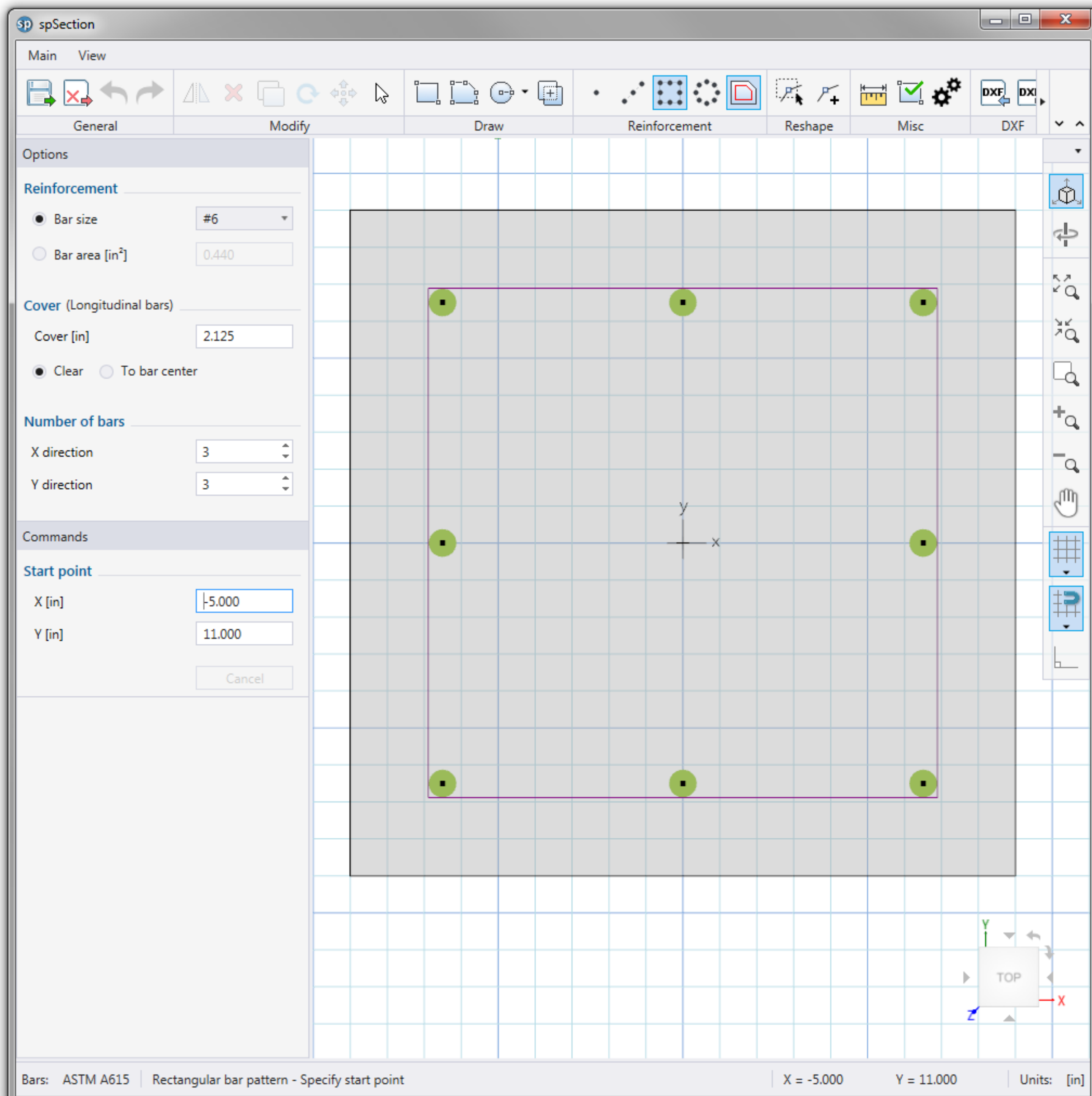



Figure 8 – spColumn Model Editor (spSection)

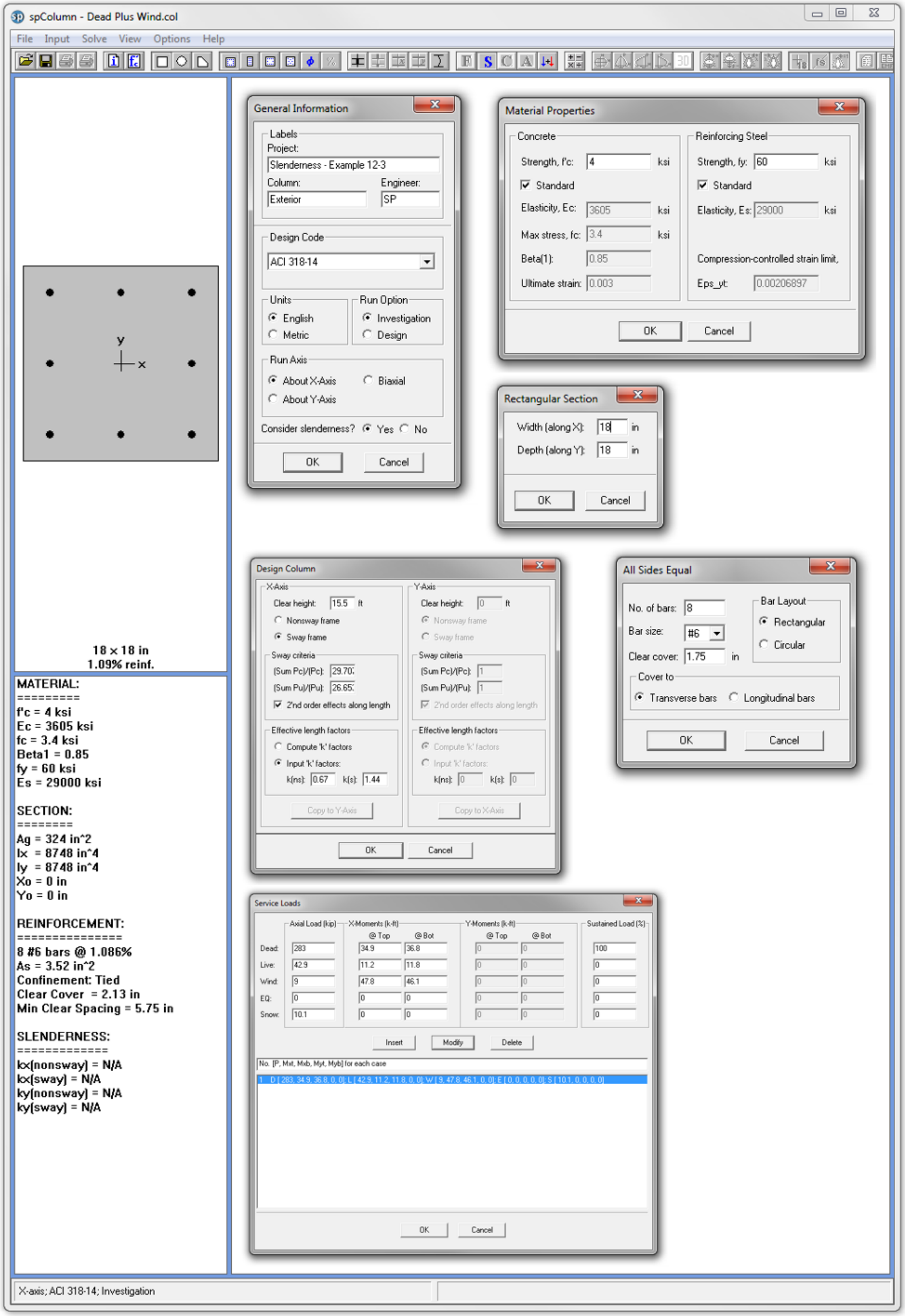



Figure 9 –spColumn Model Input Wizard Windows

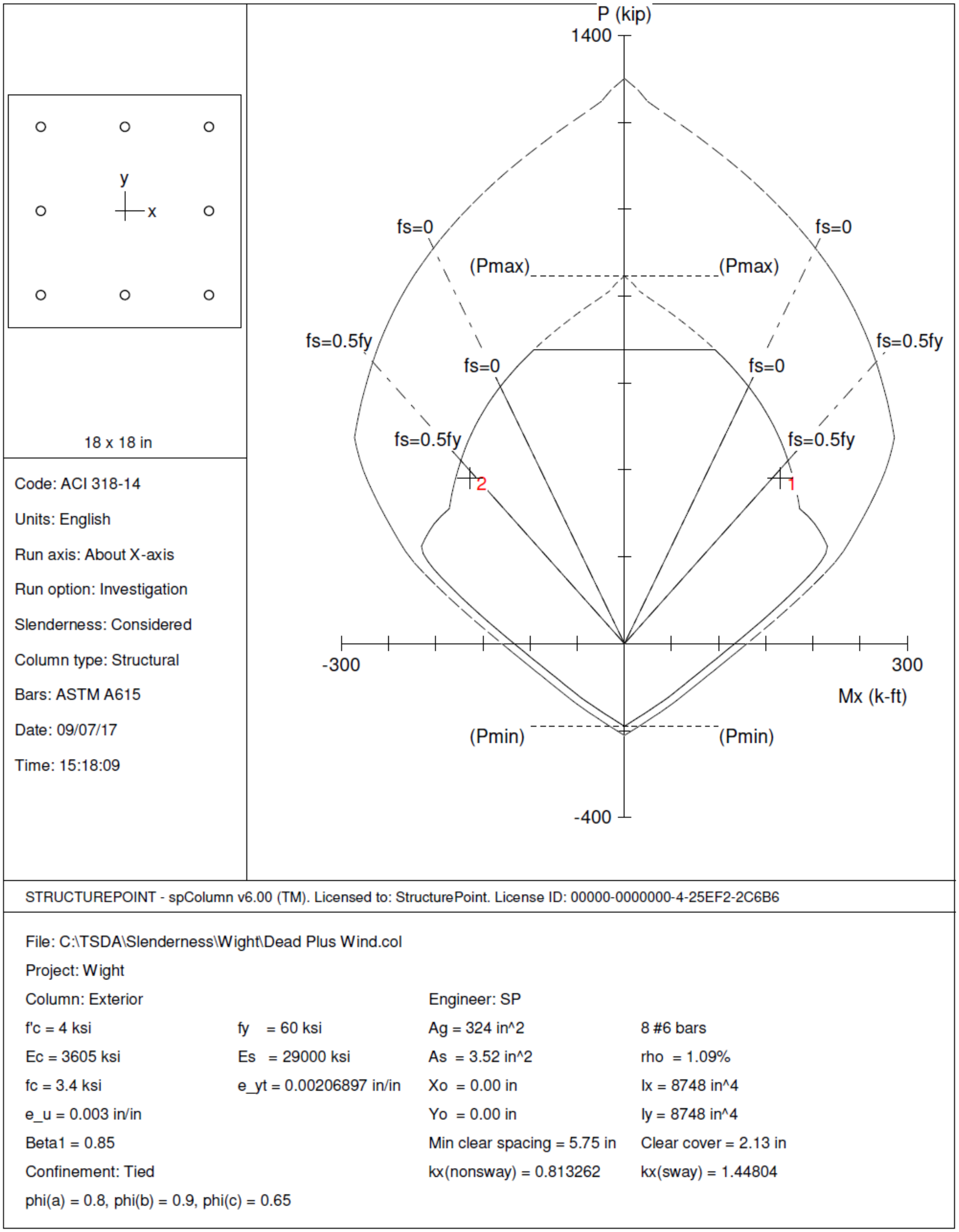
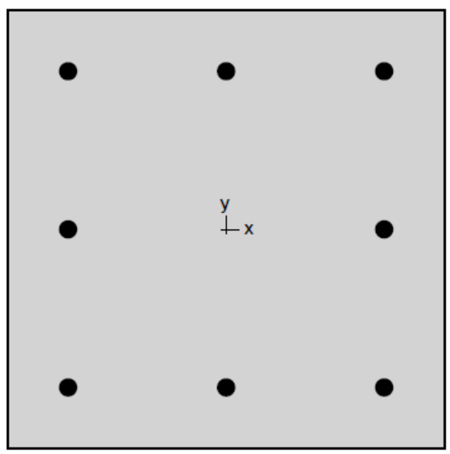



Figure 10 – Column Section Interaction Diagram about X-Axis – Design Check for Load Combination 6 (spColumn)

spColumn v6.00
Computer program for the Strength Design of Reinforced Concrete Sections

# Contents



# List of Figures

# 1. General Information

| | |
|---|---|
| File Name | C:\TSDA\Slenderness\Wight\Dead Plus Wind.col |
| Project | Wight |
| Column | Exterior |
| Engineer | SP |
| Code | ACI 318-14 |
| Bar Set | ASTM A615 |
| Units | English |
| Run Option | Investigation |
| Run Axis | X - axis |
| Slenderness | Considered |
| Column Type | Structural |

# 2. Material Properties

## 2.1. Concrete

| | | |
|---|---|---|
| Type | Standard | |
| $f'_c$ | 4 | ksi |
| $E_c$ | 3605 | ksi |
| $f_c$ | 3.4 | ksi |
| $\varepsilon_u$ | 0.003 | in/in |
| $\beta_1$ | 0.85 | |

## 2.2. Steel

| | | |
|---|---|---|
| Type | Standard | |
| $f_y$ | 60 | ksi |
| $E_s$ | 29000 | ksi |
| $\varepsilon_{yt}$ | 0.00206897 | in/in |

# 3. Section

## 3.1. Shape and Properties

| | | |
|---|---|---|
| Type | Rectangular | |
| Width | 18 | in |
| Depth | 18 | in |
| $A_g$ | 324 | $in^2$ |
| $I_x$ | 8748 | $in^4$ |
| $I_y$ | 8748 | $in^4$ |
| $r_x$ | 5.19615 | in |
| $r_y$ | 5.19615 | in |
| $X_o$ | 0 | in |
| $Y_o$ | 0 | in |

### *3.2. Section Figure*

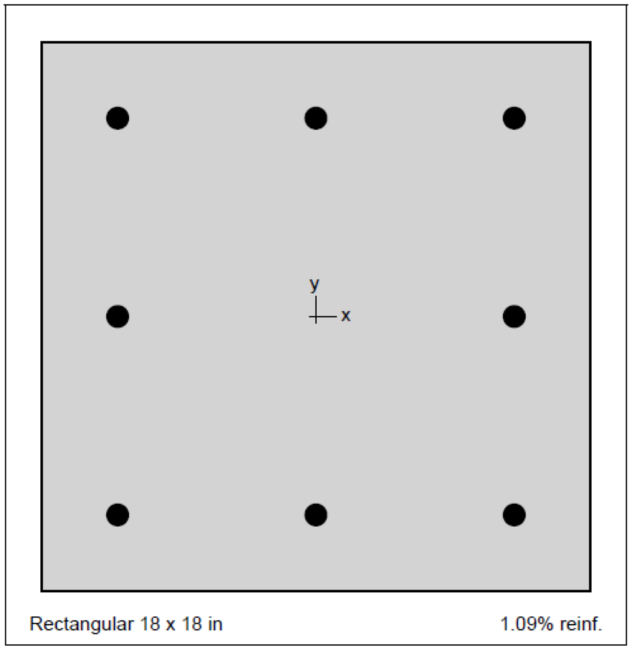



**Figure 1: Column section**

## 4. Reinforcement

### *4.1. Bar Set: ASTM A615*

| Bar | Diameter | Area | Bar | Diameter | Area | Bar | Diameter | Area |
|---|---|---|---|---|---|---|---|---|
| | in | in$^2$ | | in | in$^2$ | | in | in$^2$ |
| #3 | 0.38 | 0.11 | #4 | 0.50 | 0.20 | #5 | 0.63 | 0.31 |
| #6 | 0.75 | 0.44 | #7 | 0.88 | 0.60 | #8 | 1.00 | 0.79 |
| #9 | 1.13 | 1.00 | #10 | 1.27 | 1.27 | #11 | 1.41 | 1.56 |
| #14 | 1.69 | 2.25 | #18 | 2.26 | 4.00 | | | |

### *4.2. Confinement and Factors*

| | |
|---|---|
| Confinement type | Tied |
| For #10 bars or less | #3 ties |
| For larger bars | #4 ties |
| | |
| **Capacity Reduction Factors** | |
| Axial compression, (a) | 0.8 |
| Tension controlled failure, (b) | 0.9 |
| Compression controlled failure, (c) | 0.65 |

### *4.3. Arrangement*

| | |
|---|---|
| Pattern | All sides equal |
| Bar layout | Rectangular |
| Cover to | Transverse bars |
| Clear cover | 1.75 in |
| Bars | 8 #6 |

| | |
|---|---|
| Total steel area, $A_s$ | 3.52 in² |
| rho | 1.09 % |
| Minimum clear spacing | 5.75 in |

# 5. Loading

## 5.1. Load Combinations

| Combination | Dead | Live | Wind | EQ | Snow |
|---|---|---|---|---|---|
| U1 | 1.200 | 0.500 | 1.600 | 0.000 | 0.500 |

## 5.2. Service Loads

| No | Load case | Axial load | Mx @ Top | Mx @ Bottom | My @ Top | My @ Bottom |
|---|---|---|---|---|---|---|
| | | kip | k-ft | k-ft | k-ft | k-ft |
| 1 | Dead | 283.00 | 34.90 | 36.80 | 0.00 | 0.00 |
| 1 | Live | 42.90 | 11.20 | 11.80 | 0.00 | 0.00 |
| 1 | Wind | 9.00 | 47.80 | 46.10 | 0.00 | 0.00 |
| 1 | EQ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1 | Snow | 10.10 | 0.00 | 0.00 | 0.00 | 0.00 |

## 5.3. Sustained Load Factors

| Load case | Factor |
|---|---|
| | % |
| Dead | 100 |
| Live | 0 |
| Wind | 0 |
| EQ | 0 |
| Snow | 0 |

# 6. Slenderness

## 6.1. Sway Criteria

| | |
|---|---|
| X-Axis | Sway column |
| 2nd order effects along length | Considered |
| $\sum P_c$ | 29.71 x $P_c$ |
| $\sum P_u$ | 26.65 x $P_u$ |

## 6.2. Columns

| Column | Axis | Height | Width | Depth | I | $f'_c$ | $E_c$ |
|---|---|---|---|---|---|---|---|
| | | ft | in | in | in⁴ | ksi | ksi |
| Design | X | 15.5 | 18 | 18 | 8748 | 4 | 3605 |
| Above | X | 11.5 | 18 | 18 | 8748 | 4 | 3605 |
| Below | X | (no column specified...) | | | | | |

## 6.3. X - Beams

| Beam | Length | Width | Depth | I | $f'_c$ | $E_c$ |
|---|---|---|---|---|---|---|
| | ft | in | in | in⁴ | ksi | ksi |
| Above Left | 32 | 18 | 30 | 40500 | 4 | 3605 |
| Above Right | (no beam specified...) | | | | | |
| Below Left | 18.15 | 18.15 | 18.15 | 9043.26 | 4 | 3605 |
| Below Right | 18.15 | 18.15 | 18.15 | 9043.26 | 4 | 3605 |

# 7. Moment Magnification

## *7.1. General Parameters*

| Factors | Code defaults |
|---|---|
| Stiffness reduction factor, $\Phi_K$ | 0.75 |
| Cracked section coefficients, cI(beams) | 0.35 |
| Cracked section coefficients, cI(columns) | 0.7 |
| | |
| $0.2 E_cI_g + E_sI_{se}$ (X-axis) | 9.54e+006 kip-in² |
| Minimum eccentricity, $E_{x\,min}$ | 1.14 in |

## *7.2. Effective Length Factors*

| Axis | $\Psi_{top}$ | $\Psi_{bottom}$ | k (Nonsway) | k (Sway) | $kl_u/r$ |
|---|---|---|---|---|---|
| X | 1.992 | 1.003 | 0.813 | 1.448 | 51.83 |

## *7.3. Magnification Factors: X - axis*

| Load Combo | | At Ends | | | | | Along Length | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\Sigma P_u$ kip | $P_c$ kip | $\Sigma P_c$ kip | $\beta_{ds}$ | $\delta_s$ | $P_u$ kip | $k'l_u/r$ | $P_c$ kip | $\beta_d$ | $C_m$ | $\delta$ |
| 1 | U1 | 10141.47 | 1298.22 | 38566.28 | 0.000 | 1.540 | 380.50 | (N/A) | 2174.77 | 0.893 | 0.204 | 1.000 |

# 8. Factored Moments

NOTE: Each loading combination includes the following cases:
Top - At column top
Bot - At column bottom

## *8.1. X - axis*

| Load Combo | | | 1st Order $M_{ns}$ k-ft | $M_s$ k-ft | $M_u$ k-ft | $M_{min}$ k-ft | 2nd Order | $M_i$ k-ft | $M_c$ k-ft | Ratio 2nd/1st |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | U1 | Top | 47.48 | 76.48 | 123.96 | 36.15 | $M_2$= | 165.25 | 165.25 | 1.333 |
| 1 | U1 | Bot | -50.06 | -73.76 | -123.82 | -36.15 | $M_1$= | -163.64 | -163.64 | 1.322 |

# 9. Factored Loads and Moments with Corresponding Capacities

NOTE: Each loading combination includes the following cases:
Top - At column top
Bot - At column bottom

| No. | Load Combo | | | $P_u$ kip | $M_{ux}$ k-ft | $\Phi M_{nx}$ k-ft | $\Phi M_n/M_u$ | NA Depth in | $d_t$ Depth in | $\varepsilon_t$ | $\Phi$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | U1 | Top | 380.50 | 165.25 | 178.96 | 1.083 | 10.65 | 15.50 | 0.00137 | 0.650 |
| 2 | 1 | U1 | Bot | 380.50 | -163.64 | -178.96 | 1.094 | 10.65 | 15.50 | 0.00137 | 0.650 |

## 8. Summary and Comparison of Design Results

Analysis and design results from the hand calculations above are compared for the one load combination used in the reference (Example 12-3) and exact values obtained from spColumn model.

Table 8 – Parameters for Moment Magnification along the Column Length

| | Q | k | $\beta_{dns}$ | $C_m$ | $I_{se}$ | $P_c$, kip | $\delta$ | $M_{2(min)}$, ft-kip | $M_{2(2nd)}$, ft-kip |
|---|---|---|---|---|---|---|---|---|---|
| Hand | 0.21 | 0.81* | 0.82 | 0.221 | 111.5‡ | 2,277 | 1 > 0.264 | 39.3 | 63.04 |
| Reference | 0.21 | 1.00† | 0.82 | 0.22 | 150.0† | 1,660 | 1 > 0.330 | 39.3 | 63 |
| spColumn | --- | 0.81‡ | 0.82 | 0.221 | 111.5‡ | 2,259 | 1 > 0.270 | 39.3 | 63.04 |

* From nomographs (ACI 318 charts)
† Conservatively estimated not using exact formulae without impact on the final results in this special case
‡ Exact formulated answer

In this table, a detailed comparison for all considered load combinations are presented for comparison.

Table 9 - Factored Axial loads and Magnified Moments at Column Ends

| No. | $P_u$, kip | | $\delta_s$ | | $M_{1(2nd)}$, ft-kip | | $M_{2(2nd)}$, ft-kip | |
|---|---|---|---|---|---|---|---|---|
| | Hand | spColumn | Hand | spColumn | Hand | spColumn | Hand | spColumn |
| 1 | 396.2 | 396.2 | N/A | N/A | 48.9 | 48.9 | 51.5 | 51.5 |
| 2 | 413.3 | 413.3 | N/A | N/A | 59.8 | 59.8 | 63.0 | 63.0 |
| 3 | 377.2 | 377.2 | N/A | N/A | 47.5 | 47.5 | 50.1 | 50.1 |
| 4 | 363.0 | 363.0 | 1.51 | 1.51 | 99.6 | 99.6 | 99.9 | 99.9 |
| 5 | 348.6 | 348.6 | 1.51 | 1.51 | -33.1 | -33.1 | -33.1 | -33.1 |
| 6 | 380.5 | 380.5 | 1.53 | 1.53 | 163.0 | 163.0 | 164.5 | 164.5 |
| 7 | 351.7 | 351.7 | 1.53 | 1.53 | -62.8 | -62.8 | -69.6 | -69.6 |
| 8 | 269.1 | 269.1 | 1.32 | 1.32 | 130.4 | 130.4 | 132.2 | 132.2 |
| 9 | 240.3 | 240.3 | 1.32 | 1.32 | -64.1 | -64.1 | -69.4 | -69.4 |

| Table 10 - Factored Axial loads and Magnified Moments along Column Length | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| No. | $\delta$ | | $M_{c1}$, ft-kip | | $M_{c2}$, ft-kip | | $M_{c1}/M_{1(1st)}$ | | $M_{c2}/M_{2(1st)}$ | |
| | Hand | spColumn | Hand | spColumn | Hand | spColumn | Hand | spColumn | Hand | spColumn |
| 1 | 1.00 | 1.00 | 48.9 | 48.9 | 51.5 | 51.5 | 1.00 | 1.00 | 1.00 | 1.00 |
| 2 | 1.00 | 1.00 | 59.8 | 59.8 | 63.0 | 63.0 | 1.00 | 1.00 | 1.00 | 1.00 |
| 3 | 1.00 | 1.00 | 47.5 | 47.5 | 50.1 | 50.1 | 1.00 | 1.00 | 1.00 | 1.00 |
| 4 | 1.00 | 1.00 | 99.6 | 99.6 | 99.9 | 99.9 | 1.24 | 1.24 | 1.23 | 1.23 |
| 5 | 1.00 | 1.00 | -33.1 | -33.1 | 33.1 | 33.1 | 1.00 | 1.00 | 1.00 | 1.00 |
| 6 | 1.00 | 1.00 | 163.0 | 163.0 | 164.5 | 164.5 | 1.32 | 1.32 | 1.33 | 1.33 |
| 7 | 1.00 | 1.00 | -62.8 | -62.9 | -69.6 | -69.6 | 1.88 | 1.88 | 2.08 | 2.08 |
| 8 | 1.00 | 1.00 | 130.4 | 130.4 | 132.2 | 132.2 | 1.22 | 1.22 | 1.23 | 1.23 |
| 9 | 1.00 | 1.00 | -64.1 | -64.1 | -69.4 | -69.4 | 1.58 | 1.58 | 1.54 | 1.54 |

| Table 11 - Design Parameters Comparison | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| No. | c, in. | | $\varepsilon_t = \varepsilon_s$ | | $\varphi$ | | $\varphi P_n$, kip | | $\varphi M_n$, kip.ft | |
| | Hand | spColumn | Hand | spColumn | Hand | spColumn | Hand | spColumn | Hand | spColumn |
| 1 | 10.98 | 10.98 | 0.00123 | 0.00123 | 0.65 | 0.65 | 396.2 | 396.2 | 177.0 | 177.0 |
| 2 | 11.35 | 11.35 | 0.00110 | 0.00110 | 0.65 | 0.65 | 413.3 | 413.3 | 174.7 | 174.7 |
| 3 | 10.55 | 10.54 | 0.00141 | 0.00141 | 0.65 | 0.65 | 377.6 | 377.6 | 179.5 | 179.5 |
| 4 | 10.25 | 10.25 | 0.00154 | 0.00154 | 0.65 | 0.65 | 363.0 | 363.0 | 181.1 | 181.1 |
| 5 | 9.96 | 9.96 | 0.00167 | 0.00167 | 0.65 | 0.65 | 348.6 | 348.6 | 182.6 | 182.6 |
| 6 | 10.65 | 10.65 | 0.00137 | 0.00137 | 0.65 | 0.65 | 380.4 | 380.4 | 179.0 | 179.0 |
| 7 | 10.02 | 10.02 | 0.00164 | 0.00164 | 0.65 | 0.65 | 351.6 | 351.6 | 182.3 | 182.3 |
| 8 | 7.26 | 7.30 | 0.00340 | 0.00337 | 0.77 | 0.76 | 269.2 | 269.1 | 205.1 | 204.0 |
| 9 | 6.36 | 6.37 | 0.00431 | 0.00430 | 0.84 | 0.84 | 240.2 | 240.3 | 212.2 | 211.7 |

All the results of the hand calculations illustrated above are in precise agreement with the automated exact results obtained from the spColumn program.

## 9. Conclusions & Observations

The analysis of the reinforced concrete section performed by spColumn conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames.

ACI 318 provides multiple options for calculating values of *k*, $(EI)_{eff}$, $\delta_s$, and $\delta$ leading to variability in the determination of the adequacy of a column section. Engineers must exercise judgment in selecting suitable options to match their design condition as is the case in the reference where the author conservatively made assumptions to simplify and speed the calculation effort. The spColumn program utilizes the exact methods whenever possible and allows user to override the calculated values with direct input based on their engineering judgment wherever it is permissible.

In load combinations 7 and 9, $M_u$ including second-order effects exceeds 1.4 $M_u$ due to first-order effects (see Table 6). This indicates that in this building, the weight of the structure is high in proportion to its lateral stiffness leading to excessive $P\Delta$ effect (secondary moments are more than 25 percent of the primary moments). The $P\Delta$ effects will eventually introduce singularities into the solution to the equations of equilibrium, indicating physical structural instability. It was concluded in the literature that the probability of stability failure increases rapidly when the stability index $Q$ exceeds 0.2, which is equivalent to a secondary-to-primary moment ratio of 1.25. The maximum value of the stability coefficient $\theta$ (according to ASCE/SEI 7) which is close to stability coefficient $Q$ (according to ACI 318) is 0.25. The value 0.25 is equivalent to a secondary-to-primary moment ratio of 1.33. Hence, the upper limit of 1.4 on the secondary-to-primary moment ratio was selected by the ACI 318.

As can be seen in Table 6 of this example, exploring the impact of other code permissible equation options provides the engineer added flexibility in decision making regarding design. For load combinations 7 & 9 resolving the stability concern may be viable through a frame analysis providing values for $V_{us}$ and $\Delta_o$ to calculate magnification factor $\delta_s$ and may allow the proposed design to be acceptable by inspection of load combination #6. Creating a complete model with detailed lateral loads and load combinations to account for second order effects may not be warranted for all cases of slender column design nor is it disadvantageous to have a higher margin of safety when it comes to column slenderness and frame stability considerations.