Comparison of Effective Flexural Stiffness for Critical Buckling of Concrete Columns and Piers

A primary concern in calculating the critical axial buckling load P_c (Euler buckling load P_e in AASHTO) is the choice of the stiffness that reasonably approximates the variation in stiffness due to cracking, creep, and concrete nonlinearity. $(EI)_{eff}$ (or EI) is used in the process of determining the moment magnification at column ends and along the column length in sway and nonsway frames.

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2}$$

ACI 318-19/14 (6.6.4.4.2)
ACI 318-11 (10.10.6 (10-13))

$$P_c = \frac{\pi^2 (EI)_{eff}^{*}}{(kl_u)^2}$$

CSA A23.3-19/14/04 (10.15.3.1)

$$P_e = \frac{\pi^2 EI}{(kl_u)^2}$$

AASHTO 9th Edition (4.5.3.2.2b-5)

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1.0 \text{ (For nonsway frames)}$$

ACI 318-19/14 (6.6.4.5.2)
ACI 318-11 (10.10.6 (10-12))

$$\delta = \frac{C_m}{1 - \dfrac{P_f}{\phi_m^{**} P_c}} \geq 1.0 \text{ (For nonsway frames)}$$

CSA A23.3-19/14/04 (10.15.3.1)

$$\delta_b = \frac{C_m}{1 - \dfrac{P_u}{\phi_K^{**} P_e}} \geq 1.0 \text{ (For braced mode deflection)}$$

AASHTO 9th Edition (4.5.3.2.2b-3)

$$\delta_s = \frac{1}{1 - \dfrac{\Sigma P_u}{0.75 \times \Sigma P_c}} \geq 1.0 \text{ (For sway frames)}$$

ACI 318-19 /14 (6.6.4.6.2b)
ACI 318-11 (10.10.7.4 (10-21))

$$\delta_s = \frac{1}{1 - \dfrac{\Sigma P_f}{\phi_m \times \Sigma P_c}} \text{ (For sway frames)}$$

CSA A23.3-19/14/04 (10.16.3.2)

$$\delta_s = \frac{1}{1 - \dfrac{\Sigma P_u}{\phi_K \times \Sigma P_e}} \text{ (For unbraced mode deflection)}$$

AASHTO 9th Edition (4.5.3.2.2b-4)

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* CSA A23.3-14 and prior are using *EI* instead of *(EI)eff*.

** Where $\phi_m = 0.75$ for CSA and $\phi_K = 0.75$ for AASHTO

Design codes provide the following options to calculate $(EI)_{eff}$ as follows:

$$(EI)_{eff} = \begin{Bmatrix} \dfrac{0.4E_cI_g}{1+\beta_{dns}} & (6.6.4.4.4a) \\ \dfrac{0.2E_cI_g + E_sI_{se}}{1+\beta_{dns}}^{\dagger} & (6.6.4.4.4b) \\ \dfrac{E_cI}{1+\beta_{dns}} & (6.6.4.4.4c) \end{Bmatrix}$$

ACI 318-19/14 (6.6.4.4.4)
ACI 318-11 (10.10.6.1)

$$(EI)^{*}_{eff} = \begin{Bmatrix} \dfrac{0.2E_cI_g + E_sI_{st}}{1+\beta_d}^{\dagger} & (Eq.\ 10.19) \\ \dfrac{0.4E_cI_g}{1+\beta_d} & (Eq.\ 10.20) \end{Bmatrix}$$

CSA A23.3-19/14/04 (10.15.3.1)

$$EI = \text{Larger of} \begin{Bmatrix} \dfrac{\dfrac{E_cI_g}{5} + E_sI_s}{1+\beta_d} & (5.6.4.3\text{-}1) \\ \dfrac{\dfrac{E_cI_g}{2.5}}{1+\beta_d} & (5.6.4.3\text{-}2) \end{Bmatrix}$$

AASHTO 9th Edition (5.6.4.3)

† Utilized by spColumn.

* CSA A23.3-14 and prior are using *EI* instead of $(EI)_{eff}$.

Where:

β_{dns} – ACI definition

= ratio used to account for reduction of stiffness of columns due to sustained axial loads. Note that β_{dns} is used for non-sway frames and sway frames when slenderness effects are calculated along column length. For sway frames where slenderness effects are calculated at column ends, β_{ds} (the ratio of maximum factored sustained shear within a story to the maximum factored shear in that story associated with the same load combination) is used instead of β_{dns}.

β_d – CSA definition

= (for non-sway frames and for strength and stability checks of sway frames) the ratio of the maximum factored sustained axial load to the maximum factored axial load associated with the same load combination.

= (for sway frames) the ratio of the maximum factored sustained shear within a storey to the maximum factored shear in that storey.

β_d – AASHTO definition

= ratio of maximum factored permanent load moments to maximum factored total load moment (always positive).

For ACI, the moment of inertia of the column or wall section, I, in Eq. (6.6.4.4.4c) is calculated as follows:

$$0.35I_g \le I = \left(0.80 + 25\frac{A_{st}}{A_g}\right) \times \left(1 - \frac{M_u}{P_u h} - 0.5\frac{P_u}{P_o}\right) \times I_g \le 0.875I_g$$

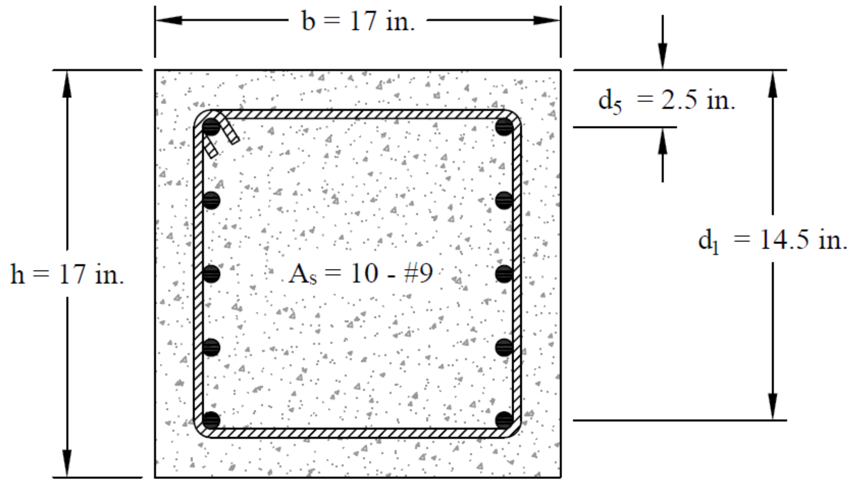
ACI 318-19/14 (Table 6.6.3.1.1(b))
ACI 318-11 (10.10.4.1 (10-8))

Comparison and Discussion

ACI 318 states that Eq. (6.6.4.4.4a) is a simplified form of Eq. (6.6.4.4.4b) and therefore, is less ‘accurate’. On the other hand, ACI 318 states that Eq. (6.6.4.4.4c) provides improved accuracy in $(EI)_{eff}$ calculation. Eq. (6.6.4.4.4c) is only provided in ACI 318.

CSA A23.3 commentary states that both Eq. (10.19) and (10.20) give approximate lower bound expressions for the effective flexural stiffness of individual compression members. Since both equations are lower bounds, it follows logically that it is appropriate to select the larger value.

Example #1



Design Data

Concrete: f_c' = 3,000 psi

Steel: f_y = 60,000 psi

Columns: h = 17 in., b = 17 in., H = 12 ft

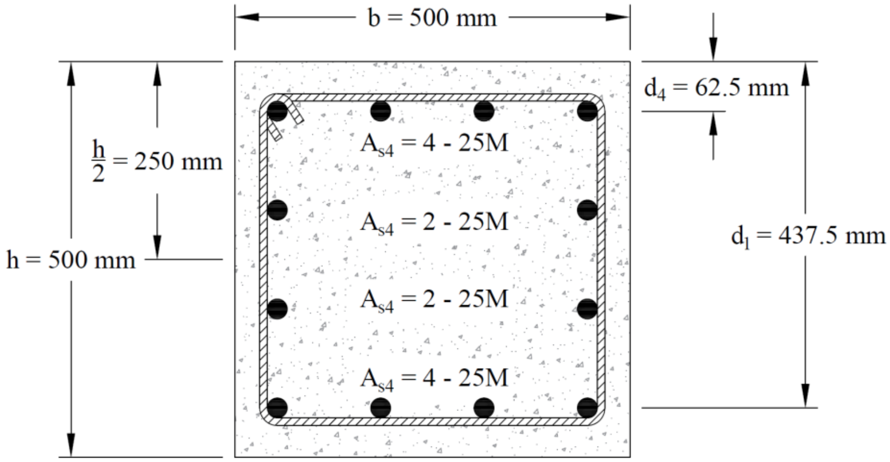
Assume 40% of the axial load is sustained.

Table 1 – Summary and Comparison Results of Example #1		
Code	**Equation**	**$(EI)_{eff}$ (kip-in.^{2})**
ACI 318-19/14/11	6.6.4.4.4b	10,561,358
CSA A23.3-19/14/04	Eq. 10-19	10,561,358
AASHTO 9th	5.6.4.3-1	10,561,358
ACI 318-19/14/11	6.6.4.4.4a	6,208,431
CSA A23.3-19/14/04	Eq. 10.20	6,208,431
AASHTO 9th	5.6.4.3-2	6,208,431
ACI 318-19/14/11	6.6.4.4.4c	13,580,943* 13,580,943**

* P_u = 525 kip, $M_{u,top}$ = 105 kip-ft, P_o = 1,311.45 kip

** P_u = 525 kip, $M_{u,bottom}$ = 0 kip-ft, P_o = 1,311.45 kip

Example #2



Design Data

Concrete: f_c' = 40 MPa

Steel: f_y = 400 MPa

Columns: h = 500 mm, b = 500 mm, H = 8.6 m

Assume 50% of the axial load is sustained.

Table 2 – Summary and Comparison Results of Example #2		
Code	**Equation**	**$(EI)_{eff}$ (kN-mm^2)**
ACI 318-19/14/11	6.6.4.4.4b	4.033×10^{10}
CSA A23.3-19/14/04	Eq. 10-19	4.033×10^{10}
AASHTO 9th	5.6.4.3-1	4.033×10^{10}
ACI 318-19/14/11	6.6.4.4.4a	4.111×10^{10}
CSA A23.3-19/14/04	Eq. 10.20	4.111×10^{10}
AASHTO 9th	5.6.4.3-2	4.111×10^{10}
ACI 318-19/14/11	6.6.4.4.4c	8.994×10^{10} * 8.994×10^{10} **
* P_u = 4,200 kN, $M_{u,top}$ = 105 kN-m, P_o = 10,696.00 kN ** P_u = 4,200 kN, $M_{u,bottom}$ = 17.5 kN-m, P_o = 10,696.00 kN		